

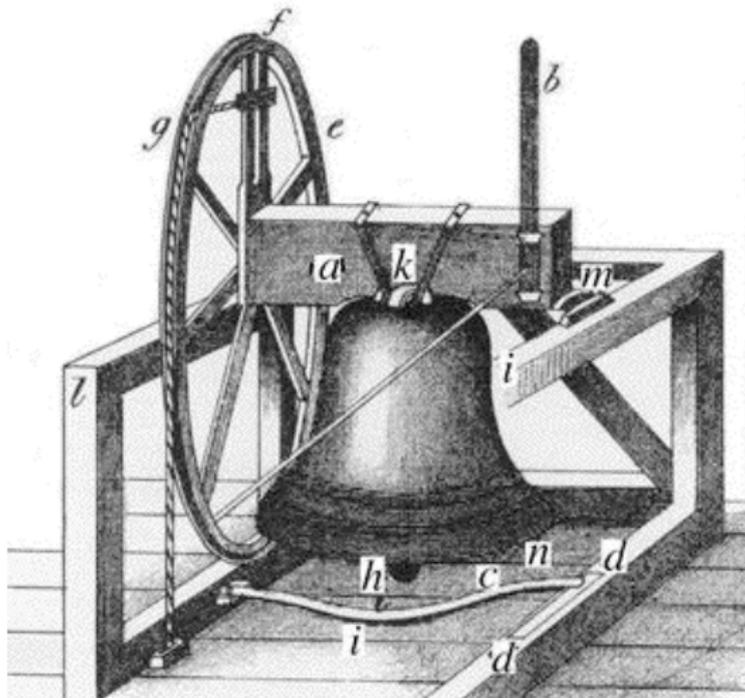
# Change Ringing, Dance and Memory: An Embodied Learning Approach to Abstract Algebra

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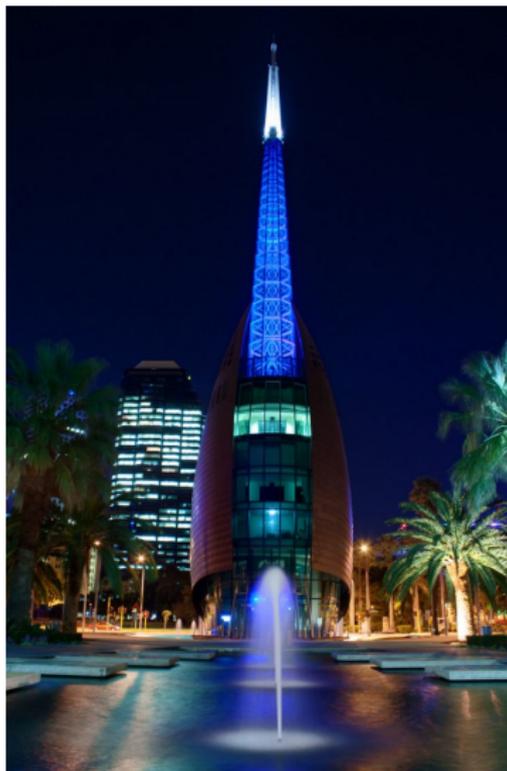


- a. Stock
- b. Stay
- c. Slider
- d. Blocks
- e. Wheel
- f. Groove of Wheel
- g. Fillet
- h. Ball of Clapper
- i. Flight of Clapper
- k. Cannons
- l. Timber of Cage
- m. Gudgeons
- n. Lip of Bell

**Figure :** A large bell and the apparatus required to make it ring.



Figure : Bell ringing demonstration in the Swan Bells Tower, Perth, Australia.



**Figure :** The Swan Bells Tower, a unique icon for Western Australia. Contains 12 royal bells from England (St. Martin-in-the-Fields).



**Figure** : The floating belfry on the Thames river, London, during the Queen's Diamond Jubilee in June, 2012. © PA.

## Change Ringing: Rules

Rules to ring an **extent** on  $n$  bells:

- 1 The first and last changes (rows) are rounds (1 2 3 4  $\dots$   $n$ ).
- 2 Other than rounds, all of the other  $n!$  changes occur exactly once.
- 3 Between successive changes, no bell moves more than one position.
- 4 No bell rests for more than 2 (sometimes relaxed further to 4) positions.
- 5 Each working bell should do the same amount of “work” (obey the same overall pattern).
- 6 A retrograde symmetry should be present in the extent to help the ringers learn the path of their respective bell. This is called the **palindrome property**.

**Note:** Rules 1 - 3 are mandatory for an extent while Rules 4 - 6 are optional, though often satisfied.

## Change Ringing: Three Bells

The two extents on three bells:

1 2 3	1 2 3
2 1 3	1 3 2
2 3 1	3 1 2
3 2 1	3 2 1
3 1 2	2 3 1
<u>1 3 2</u>	<u>2 1 3</u>
1 2 3	1 2 3

Note the simple zig-zag pattern of Bell 1 in the first extent, sweeping easily from position 1 to position 3 and back again. We say that Bell 1 is **plain hunting**; it is considered to be a “non-working” bell. Bell 3 is **hunting down** in the second extent.

## Change Ringing: Four Bells

Plain Bob Minimus (read down first, then jump to next column)

1 2 3 4	1 3 4 2	1 4 2 3
2 1 4 3	3 1 2 4	4 1 3 2
2 4 1 3	3 2 1 4	4 3 1 2
4 2 3 1	2 3 4 1	3 4 2 1
4 3 2 1	2 4 3 1	3 2 4 1
3 4 1 2	4 2 1 3	2 3 1 4
3 1 4 2	4 1 2 3	2 1 3 4
1 3 2 4	1 4 3 2	<u>1 2 4 3</u>
		1 2 3 4

Let  $a = (12)(34)$ ,  $b = (23)$ ,  $c = (34)$ . The above sequence of 24 permutations can be "factored" as

$$[(ab)^3 ac]^3 = [abababa c]^3 \quad \text{Palindrome!}$$

## Math, Music and Memory

- Special interdisciplinary first-year seminar taught as part of a college-wide program called **Montserrat**.
- **Arts Transcending Borders**: Funded by a grant from the Andrew W. Mellon Foundation, ATB strives to promote and integrate the arts across campus. Program hosts visiting artists and performers who work directly with faculty and students.
- **Liz Lerman**—award-winning choreographer, educator, and McArthur “Genius Grant” recipient. Has worked with a wide variety of groups ranging from the University of Maryland Symphony Orchestra to biologists at Wesleyan University.
- **Embodied learning** is using movement to creatively teach concepts at a more fundamental level in order to improve student comprehension and retention. Having students reflect during the learning process is a key feature.

## The Class

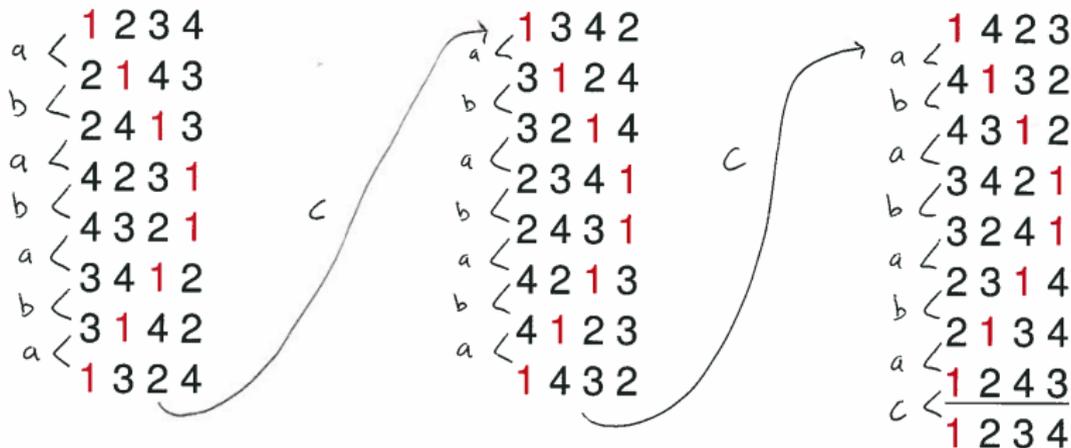
**Goal:** Use embodied learning principles to teach some of the mathematical ideas inherent in change ringing (permutations, combinatorics, group theory).

**Activity:** Students were broken up into quartets (17 students = 4 quartets) and assigned the task of memorizing and performing all 25 rows of Plain Bob Minimus using hand bells.



## Plain Bob Minimus

Read down first, then move to the next column.



**Permutations:** Let  $a = (12)(34)$ ,  $b = (23)$ ,  $c = (34)$ . The above sequence of 24 permutations can be "factored" as

$$[(ab)^3 ac]^3 = [abababa c]^3 \quad \text{Palindrome!}$$

## Survey Results: Sample Student Comments

- “The approach worked very well for me. Being interactive with the actual bells made it easier to memorize the piece.”
- “Before, I understood [that] ‘a means these 2 switch,’ but once we actually switched positions, it all made much more sense.”
- “It is not that I **learned** more; it is that I **understood** more.”
- “Liz seemed to be less focused on the actual mission of playing the song correctly than on the pieces we learned, and what we got from the experience, which was good. For her, it was more about the journey than the destination.”
- “I noticed that even though we weren’t playing the bells for a **grade**, we were all **eager** to play them correctly.”
- 16/17 students were able to recall the first 7 changes of *Plain Bob* and 10/17 knew the entire first lead. One student correctly recalled the entire piece (even though it wasn’t asked for!).

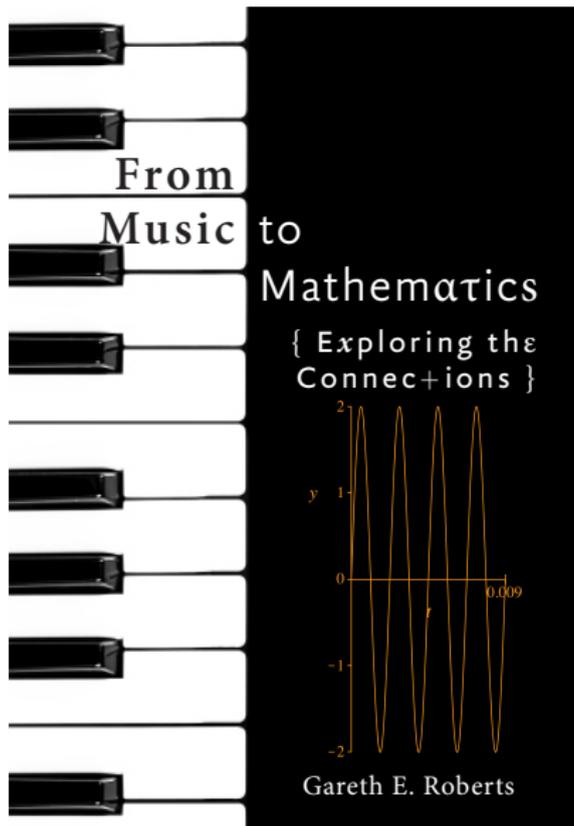


Figure : Textbook coming out soon (Johns Hopkins University Press).