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*Math, Music and Identity*
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Béla Bartók

- Born in Nagyszentmiklós Hungary (now Sînnicolau Mare, Romania) in 1881. Died in New York, Sept. 1945.

- Could play 40 songs on the piano by age 4. Wrote first piece of music at age 6. Quickly became a chapel organist and an accomplished pianist and composer.

- Studied at the Catholic Gymnasium (high school) in Pozsony where he excelled in math and physics in addition to music. Entered the Academy of Music (Liszt is 1st president) in Budapest in 1899.

- Avid collector of folk music (particularly Hungarian, Romanian, Slovakian, and Turkish).

- Influenced by Debussy and Ravel; preferred Bach to Beethoven.

- Considered to be one of Hungary’s greatest composers (along with Franz Liszt).
Béla Bartók (cont.)

Figure: Bartók at age 22.


- “We follow nature in composition ... folk music is a phenomenon of nature. Its formations developed as spontaneously as other living natural organisms: the flowers, animals, etc.” — Bartók, *At the Sources of Folk Music* (1925)

- Notoriously silent about his own compositions. “Let my music speak for itself, I lay no claim to any explanation of my works!”
In 1955, the Hungarian musical analyst Ernö Lendvai started to publish works claiming the existence of the Fibonacci numbers and the golden ratio in many of Bartók’s pieces.

Some find Lendvai’s work fascinating and build from his initial ideas; others find errors in his analysis and begin to discredit him. Lendvai becomes a controversial figure in the study of Bartók’s music.

Lendvai draws connections between Bartók’s love of nature and “organic” folk music, with his compositional traits. He takes a broad view, examining form (structure of pieces, where climaxes occur, phrasing, etc.) as well as tonality (modes and intervals), in discerning a substantial use of the golden ratio and the Fibonacci numbers.
Lendvai’s analysis states:

1. Piece is 89 measures long.
2. The climax of the movement occurs at the end of bar 55 (loudest moment), which gives a subdivision of two Fibonacci numbers (34 and 55) that are an excellent approximation to the golden ratio.
3. Violin mutes begin to be removed in bar 34 and are placed back on in bar 69 \((56 + 13 = 69)\).
4. The exposition in the opening ends after 21 bars.
INSTRUMENTATION

Strings to be placed in two different groups

1st Group: Violins I and II, Viola I
Violoncello I, Double Bass I

2nd Group: Violins III and IV, Viola II
Violoncello II, Double Bass II

Side Drum without snares,
Side Drum with snares
*Cymbals
Tam-Tam
Bass Drum
**Timpani (chromatic)
Xylophone
Celesta (takes over at certain places the second part
of the piano)
Harp
Pianoforte

APPROXIMATE POSITION OF THE ORCHESTRA

<table>
<thead>
<tr>
<th>Violoncello I</th>
<th>Double Bass I</th>
<th>Double Bass II</th>
<th>Violoncello II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Timpani</td>
<td>Bass Drum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Viola I</td>
<td>Side Drums</td>
<td>Cymbals</td>
<td>Viola II</td>
</tr>
<tr>
<td>Violin II</td>
<td>Celesta</td>
<td>Xylophone</td>
<td>Violin IV</td>
</tr>
<tr>
<td>Violin I</td>
<td>Pianoforte</td>
<td>Harp</td>
<td>Violin III</td>
</tr>
</tbody>
</table>

DURATION: 26 Minutes

1st Mov.: 6 Min. 30 sec.     3rd Mov.: 6 Min. 35 sec.
2nd Mov.: 6 Min. 55 sec.     4th Mov.: 5 Min. 40 sec.

This work, completed in September 1936, was first performed at Bâle on 21st January 1937, conducted by Paul Sacher.

GENERAL REMARKS

(1) All glissandi marked ↓↑ etc. in the string, as well as in the timpani parts, are to be played in such a manner that the starting note is left immediately, so that an even gliding sound during the full value of the first note is produced.

(2) The 4th movement can, in special circumstances, be played a little slower for acoustic reasons.

* 2 pairs, one of which should be of smaller size (sounding higher).

** If no chromatic Timpani available, the Timpani glissandi should be played on the piano in corresponding chromatic passages. For this purpose an extra player, not instrument, is required.
Problems with Lendvai’s Analysis (Roy Howat)

1. The piece is 88 bars long, not 89! Lendvai includes a footnote: “The 88 bars of the score must be completed by a whole-bar rest, in accordance with the Bülow analyses of Beethoven.” Hanh?!

2. The dynamic climax of the piece is certainly at the end of bar 55. But the tonal climax is really at bar 44, when the subject returns a tritone away from the opening A to E♭. (88/2 = 44, symmetry?)

3. The viola mutes come off at the end of bar 33 (not 34). The violin mutes are placed back on at the end of bar 68 (not 69). This last fact actually helps the analysis since 68 = 55 + 13, giving the second part of the movement a division of 13 and 20 (21 if you allow the full measure rest at the end).

4. The fugal exposition actually ends in bar 20, not 21.

5. What about the celesta? It enters after bar 77. 77 − 55 = 22 (close to Fibonacci). Lendvai neglects to mention this key feature.
Figure: Roy Howat's analysis of Lendvai's work, from "Bartók, Lendvai and the Principles of Proportional Analysis," Music Analysis, 2, No. 1 (March, 1983), pp. 69-95.
Key Features of Movement I

- The piece is a fugue (think Bach) with the opening subject played by the viola starting on A. This chromatic theme only ranges a tritone in distance (6 halfsteps) to E♭.

- The successive entrances of the main theme alternate between ascending perfect 5ths (A → E → B → F♯ etc.) and descending perfect 5ths (A → D → G → C etc.). This serves to keep each subject musically “close,” as demonstrated by the circle of fifths. The themes come back to the same note a tritone away, on E♭ at the tonal climax of the movement (bar 44), exactly half way through the piece.

- In the second half of the piece (after the dynamic climax), the subject is inverted (exactly) and moves back around the circle of fifths to return to the opening A. Often, only the opening 5 notes are used (e.g., measure 65). The original 5-note opening of the subject returns in measure 82, dividing the coda (defined by the entrance of the celesta) into 4 + 7 bars.
Key Features of Movement I (cont.)

- The opening four bars (where the main theme is announced) are subdivided into 3’s and 2’s. For example, $3 + 3 + 2$ in measure 1 and $3 + 3 + 3 + 3$ in measure 2.

- The first **stretto** in the Fugue (where the initial subject is interrupted by another entrance of the subject before completing), occurs just before the end of measure 26 on the pitches $F\#$ (1st) and then C (2nd). These are precisely $1/2$ way around the circle of fifths. These return in inversion in the second half of the movement, ending in measure 68, giving a golden section split of $42 : 26$. 

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A Magnificent Inversion

A dramatic and revealing exact inversion based on the scale of the main theme occurs at the end of the first movement.

- Top part is first violins; bottom part is second violins. All other instruments are silent.
- The inversion is about A, reaffirming it as the tonal center of the movement. The motion from A to $E^b$ and back to A recaps the tonal structure of the fugue. Key idea: symmetry.
- Who was the master of using inversions in fugues? Bach!
- Last four notes: C B$\natural$ B$^b$ A, which translates in German (Bach’s native tongue) to C H B A. Coincidence?
Fig. 5: Fugue from Music for Strings, Percussion and Celeste

Dynamic arch
- 20 first episode
- 33 mutes off
- 55 climax
- 68 mutes on
- 88 end

Tonal symmetries
- 44 subject reaches Eb
- 11

Interactions
- 44 melodic peak (2nd vlns)
- 55 basses' melodic peak
- 42

Opening sequence
- 4 first melodic peak
- 44 Eb

Other connections
- 26 C/F♯ stretto
- 68 end of inverted C/F♯ stretto
- 30 end of exposition
- 77 coda begins

MUSIC ANALYSIS 2:1, 1983
Opening xylophone solo has the rhythmic pattern

1, 1, 2, 3, 5, 8, 5, 3, 2, 1, 1,

with a crescendo followed by a decrescendo (hairpin) climaxing at the top of the sequence. Obvious nod to Fibonacci as well as a nice use of retrograde symmetry.

Music for this movement famously used by Stanley Kubrick in his film adaptation of Stephen King’s *The Shining*.
Howat’s analysis of the third movement suggests a greater connection to the Fibonacci numbers and the golden ratio than in the first movement. When counting by the number of quarter notes (assuming $\frac{4}{4}$ time), the piece has 89 measures and a major subdivision into 34- and 55-measure sections.
Ex. 2: Facsimile of recto pages 1 and 2 from manuscript 80FSS1 in the New York Béla Bartók Archive, reproduced by kind permission of Dr Benjamin Suchoff, Trustee of the Bartók Estate.

\[ \gamma = 3 + 4 \cdot 4 + 3 \]

\[ \frac{54 \times 3}{162} \]

\[ \begin{array}{c}
\text{Figure: If you dig deep enough ... Bartók’s analysis of a Turkish folk song showing the Lucas numbers!}
\end{array} \]
Other Composers’ Influence on Bartók


- Kodály befriends Bartók around 1905-1906. They bond over their mutual interest in folk music (Kodály was collecting phonograph cylinders of folk music in the remote areas of Hungary).

- In 1907, Kodály writes *Méditation sur un motif de Claude Debussy*. Just as with the fugue from Bartók’s *Music for Strings, Percussion and Celesta*, the piece opens *pp* and ends *ppp*, with a central climax marked *fff*. If one counts quarter notes rather than measures, there are 508 beats. The golden ratio of 508 is 314 (to the nearest integer) and this just happens to be smack in the middle of the two climatic bars at *fff*. 
Claude Debussy (1862-1918)

- As Kodály was bringing Debussy to Bartók’s attention, Debussy composes some interesting piano pieces whose form demonstrates the golden ratio.

- *Images*, published in 1905, consists of three piano pieces: *Reflets dans l’eau*, *Mouvement* and *Hommage à Rameau*. These soon became part of Bartók’s piano repertoire.

- *Reflets* and *Mouvement* begin *pp* and finish *ppp* or *pp*, respectively. They also have main climaxes at *ff* and *fff*, respectively, located at places that divide the total piece into two portions in the golden ratio.

- *Hommage à Rameau* has a similar structure dynamically and, according to Roy Howat’s analysis, “is built very clearly on Fibonacci numbers.”
Reflets dans l’eau, Debussy

Reflets dans l’eau, Tonal Structure (Howat)

Fig. 3.3

large-scale

42 principal tonal departure

34 intermediate tonic return

34

68 final return to $\flat_7$ $\flat_7$

26

end

94

smaller-scale

16 first tonal departure

26 principal tonal departure

16

58 climactic focus $f f$ plus

10

94

return to tonic chord

76

Fig. 3.4

diatonic sequence

34 tonic

34

55 dominant

68

76 tonic

8

21

13

8

8

5

42

47
Fig. 10.4: ‘Hommage à Rameau’
Some Final Remarks on the Bartók Controversy

- Lendvai’s inaccuracies partly due to a narrow focus on the Fibonacci numbers. It’s clear that the **Lucas numbers** were more significant in the first movement of *Music for Strings, Percussion and Celesta*.

- **Strength of first movement lies in its use of symmetry:**
  1. Tonal climax in measure 44, half way through piece.
  2. Inverting the subject exactly after the climax in measure 55.
  3. Tonal symmetry built around A; mirrored trip around circle of fifths.
  4. Wonderful exact inversion at the end of the piece.
Other works by Bartók where the golden ratio can be detected are *Sonata for Two Pianos and Percussion*, *Miraculous Mandarin*, and *Divertimento*.

Bartók was highly secretive about his works. Surviving manuscripts of many of the pieces where the golden ratio appears to have been used contain no mention of it.

Bartók was already being criticized for being too “cerebral” in his music. Identifying the mathematical patterns in structure and tonality (even to his students!) would only have added fuel to the fire.

**Bottom line:** Plenty of evidence in support of mathematical ideas in *Music for Strings, Percussion and Celesta*, but don’t fudge the analysis!