Musical Group Theory

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Math/Music: Aesthetic Links
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Symmetry Operations in Music

How to get more music out of a little motif:

Translations (shifting graph vertically) \iff Transpositions (shifting notes up or down)

Example: Ballpark Music

Vertical Reflection (symmetry between right and left) \iff

Retrograde (music same forward and backward)

Example: Lean on Me

Horizontal Reflection (symmetry between top and bottom) \iff Inversion (what goes up, must come down)

Example: Bach, Bach and more Bach

Symmetry in Music: Retrograde - Haydn



Figure: Joseph Haydn, Piano Sonata No. 41, Hob. XVI/26, Minuet

Symmetry in Music: Retrograde – Bach



Figure: Johann Sebastian Bach, A Musical Offering

Symmetry in Music: Retrograde and Transposition



George F. Handel, *Messiah*, *Hallelujah chorus* (loose retrograde – form of tone painting)

The opening minute of the piece features just two motifs, the famous "Hallelujah" motif and the excerpt above.

Symmetry in Music: Inversion



John Philip Sousa, opening of the march *The Thunderer* – approximate inversion



Béla Bartók, *Mikrokosmos, No. 141, Subject and reflection* – exact inversion

Symmetry in Music: Retrograde-inversion

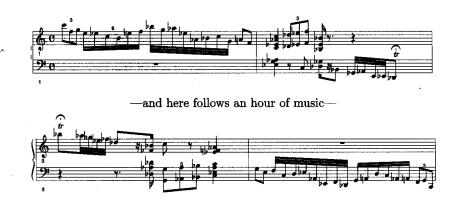
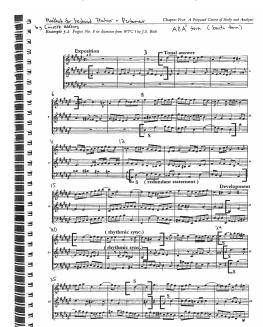


Figure: Paul Hindemith, *Ludus Tonalis* ("Tonal Game"), beginning and end. The ending Postludium is an exact retrograde-inversion (180° rotation) of the opening Praeludium.

Bach: The Well-Tempered Clavier, Fugue No. 8 in D# minor



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Combining Symmetries – Liszt



Figure: Franz Liszt, excerpt from Hungarian Rhapsody #2

Combining Symmetries – Gershwin



"I Got Rhythm" has an AABA structure, and a two-bar tag at the end. We call these four equal sections A, A, B, and A. Three of the sections are the same, and one is different. Listen to the first section, and you'll be able



Figure: George Gershwin, *I Got Rhythm*, (transposition, retrograde and inversion, all in one song!)

Der Spiegel (The Mirror) Duet W.A. Mozart Public Domain. Sequenced by Fred Nachbaur using NoteWorthy Confused? Try playing this from opposite sides of a table. Allegro

(Note: the attribution to Mozart is dubious)

Figure 9.6.

