

Musical Group Theory

Gareth E. Roberts

Department of Mathematics and Computer Science
College of the Holy Cross
Worcester, MA

Math/Music: Aesthetic Links
Montserrat Seminar Spring 2012
January 27 and 30, 2012

Symmetry Operations in Music

How to get more music out of a little motif:

Translations (shifting graph vertically) \iff Transpositions (shifting notes up or down)

Example: **Ballpark Music**

Vertical Reflection (symmetry between right and left) \iff
Retrograde (music same forward and backward)

Example: **Lean on Me**

Horizontal Reflection (symmetry between top and bottom) \iff
Inversion (what goes up, must come down)

Example: **Bach, Bach and more Bach**

Symmetry in Music: Retrograde – Haydn



Figure: Joseph Haydn, *Piano Sonata No. 41, Hob. XVII/26*, Minuet

Symmetry in Music: Retrograde – Bach



Figure: Johann Sebastian Bach, *A Musical Offering*

Symmetry in Music: Retrograde and Transposition



Symmetry in Music: Inversion



John Philip Sousa, opening of the march *The Thunderer* – approximate inversion



Béla Bartók, *Mikrokosmos*, No. 141, *Subject and reflection* – exact inversion

Symmetry in Music: Retrograde-inversion



—and here follows an hour of music—



Figure: Paul Hindemith, *Ludus Tonalis* (“Tonal Game”), beginning and end. The ending Postludium is an **exact** retrograde-inversion (180° rotation) of the opening Praeludium.

Bach: The Well-Tempered Clavier, Fugue No. 8 in D minor

Handbook for Keyboard Teacher + Performers

Chapter Five: A Proposed Course of Study and Analysis

b3 Laurette Adberg

Example 5.2: Fugue No. 8 in d minor from WTC I by J.S. Bach

ABA' form (Sonata-form)

Exposition

3 Tonal answer

6 8

11 12a

15 (redundant statement)

Development

20 (rhythmic sync.)

24 (rhythmic sync.)

25 S S

30 Inverted subject

35 36

40 (Inverted and ornamented opening)

45 S-inverted Inverted rhythmic modification

50 S 1/2 S 1/2 S 1/2 S 1/2

55

60

62a

Recapitulation

chrom. variation

S-augmented

64

S-inverted

F# Major

S-aug.

66

S

70

Coda(stretto)

S-aug.

72

S

76

rhythmic orn.

(orn.) S

(orn.)

80

84

Combining Symmetries – Liszt



Figure: Franz Liszt, excerpt from *Hungarian Rhapsody #2*

Combining Symmetries – Gershwin

9a.



**"I Got Rhythm" has
an AABA structure, and
a two-bar tag at the end.**

We call these four equal sections A, A, B, and
A. Three of the sections are the same, and one is dif-
ferent. Listen to the first section, and you'll be able

9b.

Figure: George Gershwin, *I Got Rhythm*, (transposition, retrograde and inversion, all in one song!)

Der Spiegel (The Mirror) Duet

VOCAL *Allegro* ♩=120

W.A. Mozart

mf

Allegro

Public Domain. Synchronized by Fred Nachbauer using NoteWorthy
 Condensed? Try playing this from opposite sides of a table.

(Note: the attribution to Mozart is dubious)

Figure 9.6.