# Math/Music: Aesthetic Links

## Homework Assignment #2

### DUE DATE: Fri., February 17, start of class.

Homework should be turned in at the beginning of class. You should write up solutions neatly to all problems, making sure to show all your work. You are encouraged to work on these problems with other classmates, although the solutions you turn in should be your own work. Please cite any references (web based or text) that you may have used for assistance with the assignment.

**Note:** Please list the names of any students or faculty who you worked with on the assignment.

> *Change ringing is a non-competitive and non-violent team activity that is highly stimulating intellectually and mildly demanding physically, and makes a beautiful sound. It develops mental and physical skills in a context of communal effort. The intense concentration required brings euphoric detachment that cleanses the mind of the day's petty demands and frustrations.* North American Guild of Change Ringers
> (http://www.nagcr.org/pamphlet.html)

1. Read Chapter 7 of the course text, *Ringing the changes: bells and mathematics* by Dermot Roaf and Arthur White. As you read, it is useful to follow the computations in the text to further your understanding. Some of this material was covered in class. **Side note:** During my undergraduate days at Oberlin I spent a January term (winter-term) at Western Michigan University studying change ringing with Arthur White, an Oberlin alumnus.

2. How many different ways can 10 numbered bells be arranged so that the first bell is always Bell 1 and the last bell is always Bell 10, but all other bells are arbitrarily arranged?

3. Recall that for $n = 3$ bells, there are only two allowable "moves" (permutations) between changes, interchanging the first two bells $(1\,2)$ or interchanging the last two bells $(2\,3)$. The permutation $(1\,3)$ is not allowed because bells cannot move more than one position between changes. For $n = 4$ bells, there are precisely four allowable moves: $(1\ 2), (2\ 3), (3\ 4)$ and $(1\,2)(3\,4)$. Any other permutation would move a bell more than one position away. How many allowable moves are there for $n = 5$ bells? List all of them in permutation form (e.g., $(1\,2)$).

4. How many allowable moves are there for $n = 6$ bells? List all of them in permutation form (e.g., $(1\,2)$).

5. By looking for a pattern in your answers to the previous two problems, make a table with the number of allowable moves for $n$ bells. The first column should be the number of bells $n$ and the second column should be the number of allowable moves. Begin with $n = 2$ and 1 move, then $n = 3$ and 2 moves, $n = 4$ and 4 moves, etc. The last row of your table should be $n = 12$. Describe the pattern you found to fill out the table.

6. Finish completing the group multiplication table (Table 2, last page of the *Change Ringing* handout) for the first lead of *Plain Bob Minimus*. **Hint:** Instead of doing each multiplication out by hand, try using some of the identities discussed in class on page 3 of the *Change Ringing* handout. This should save you some time. How does this table compare with the table you constructed for the dihedral group $D_4$, the symmetries of the square?

7. Your completed Table 2 should show that the set of permutations from the first lead of *Plain Bob Minimus* (call it Set $A$) are closed under multiplication of permutations. Associativity follows since the larger group $S_4$ has associativity and $e$ = rounds (the identity element) is the first change in the lead. List the inverse of each of the eight elements of the first lead and conclude that Set $A$ is indeed a subgroup of order 8 of $S_4$. In other words, Set $A$ is a group all by itself.

8. **Multiplication of permutations:** For this problem let

$$a = (1\,3\,5\,2\,4\,6) \quad \text{and} \quad b = (2\,4\,6\,1\,3\,5)$$

be two permutations in $S_6$. Compute the following:

   **a.** $a * b$

   **b.** $b * a$

   **c.** $a^2$

   **d.** $a^4$

   **e.** $b^{-1}$

For the remaining questions 9–13, let $a = (1\,2)(3\,4), b = (2\,3), c = (3\,4), d = (1\,2)$ represent the four allowable moves for an extent on $n = 4$ bells. Recall that *Plain Bob Minimus* can be written in terms of these moves as $[(ab)^3ac]^3 = [abababa\,c]^3$.

9. What is the set of eight moves $(abcd)^2$ equivalent to? Why couldn't this be used as part of an extent?

10. Referring to the work we did in class on *Canterbury Minimus* (or if you missed class on 2/10/12, you should first answer the questions at the bottom of page 2 of the *Change Ringing* handout), what are two similarities and two differences between *Canterbury Minimus* and *Plain Bob Minimus*?

11. Write out the first 25 rows of the extent on $n = 4$ bells determined by $[dbadabd\,c]^3$. This extent is called *St. Nicholas Minimus*. Does Bell 1 go *plain hunting*? Which of the six rules for an extent are satisfied?

12. Write out the first 25 rows of the extent on $n = 4$ bells determined by $[(db)^2da]^4$. This extent, which has four leads instead of three, is called *Erin Minimus*. Does Bell 1 go *plain hunting*?

13. **You are the composer!** Make up your own extent, satisfying at least the first three rules, on $n = 4$ bells that is different from *Plain Bob Minimus*, *Canterbury Minimus*, *St. Nicholas Minimus*, *Erin Minimus* or *Reverse Bob Minimus*. List all 25 rows as well as the factored permutation form of your extent. Try to make Bell 1 go plain hunting in your composition. Does your extent satisfy Rule 6, the *palindrome property*?