Math/Music: Aesthetic Links

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Musical Group Theory
Symmetry in Music
February 9 and 11, 2011

Symmetry in Music: Group Theory

How to get more music out of a little motif:

Translations (shifting graph vertically) $\Longleftrightarrow$ Transpositions (shifting notes up or down)

Ex: Ballpark Music

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Vertical Reflection (symmetry between right and left) $\Longleftrightarrow$ Retrograde (music same forward and backward)
Ex: Lean on Me

Symmetry in Music: Group Theory

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Ex: Ballpark Music

Vertical Reflection (symmetry between right and left) $\Longleftrightarrow$ Retrograde (music same forward and backward)
Ex: Lean on Me

Horizontal Reflection (symmetry between top and bottom) $\Longleftrightarrow$ Inversion (what goes up, must come down)
Ex: Bach, Bach and more Bach

Symmetry in Music: Retrograde

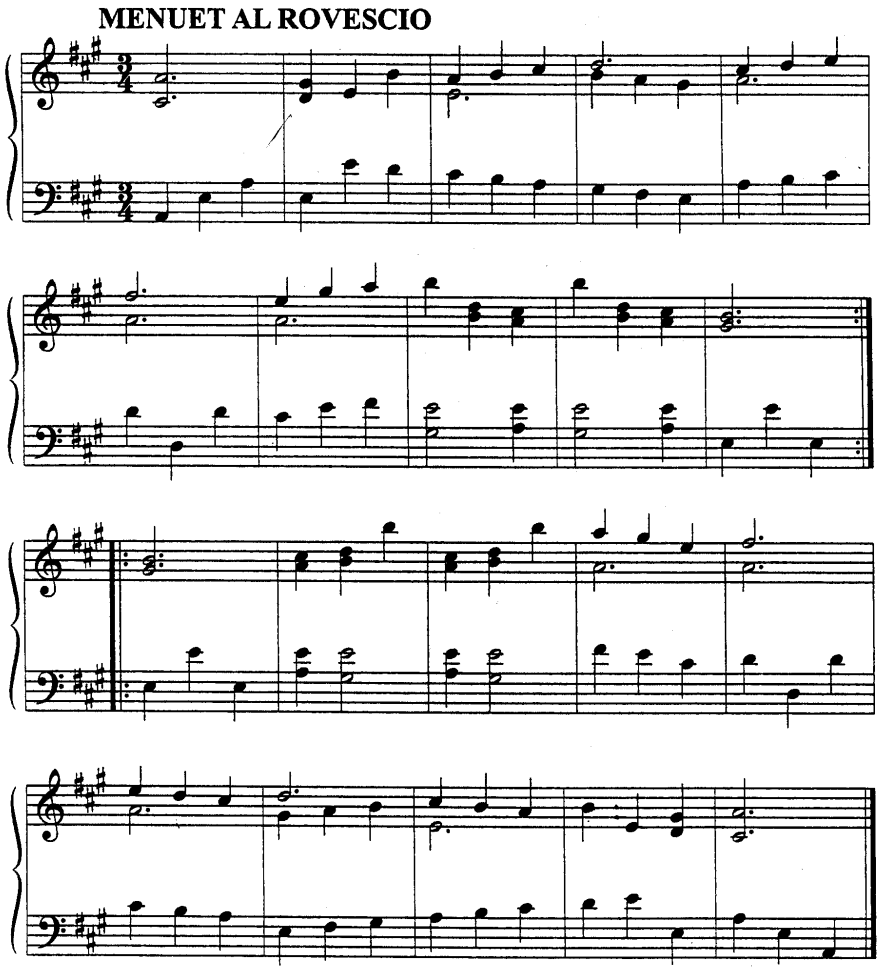


Figure: Joseph Haydn, *Piano Sonata No. 41, Hob. XVI/26*, Minuet

Symmetry in Music: Retrograde

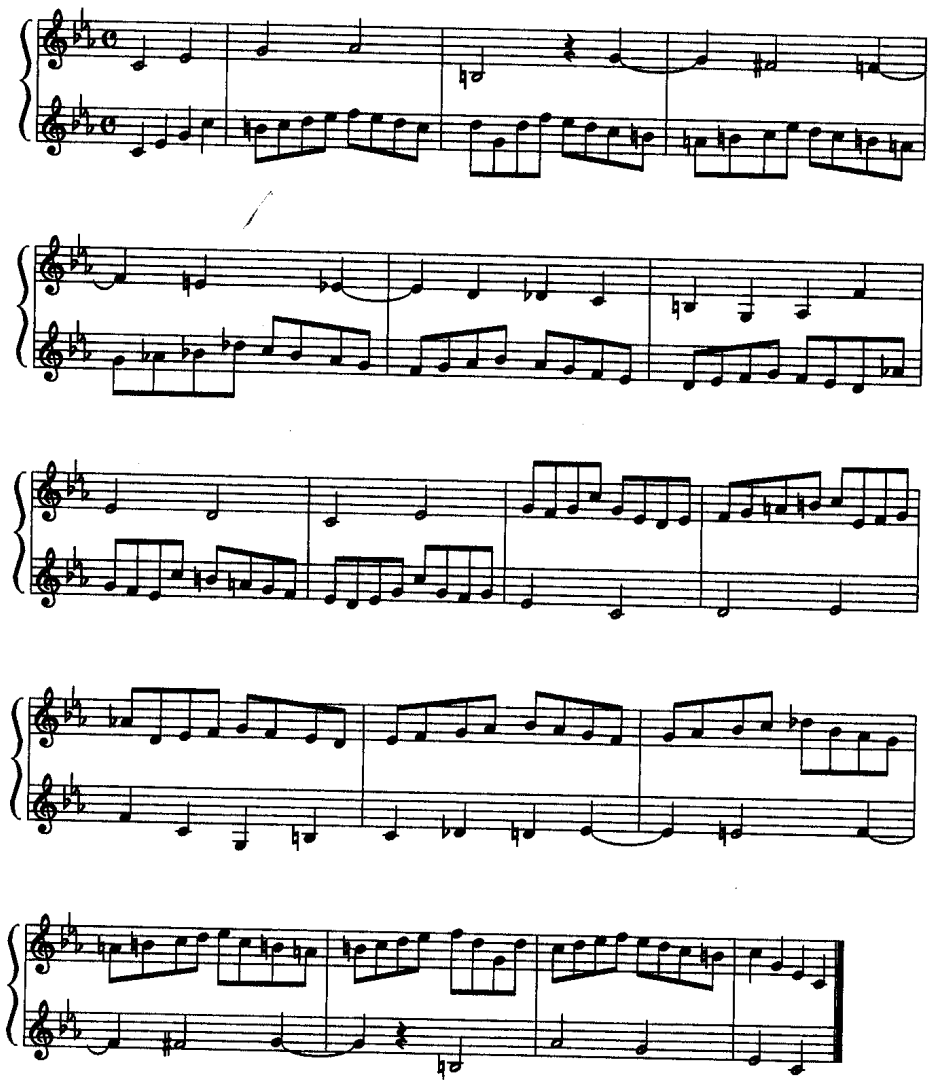
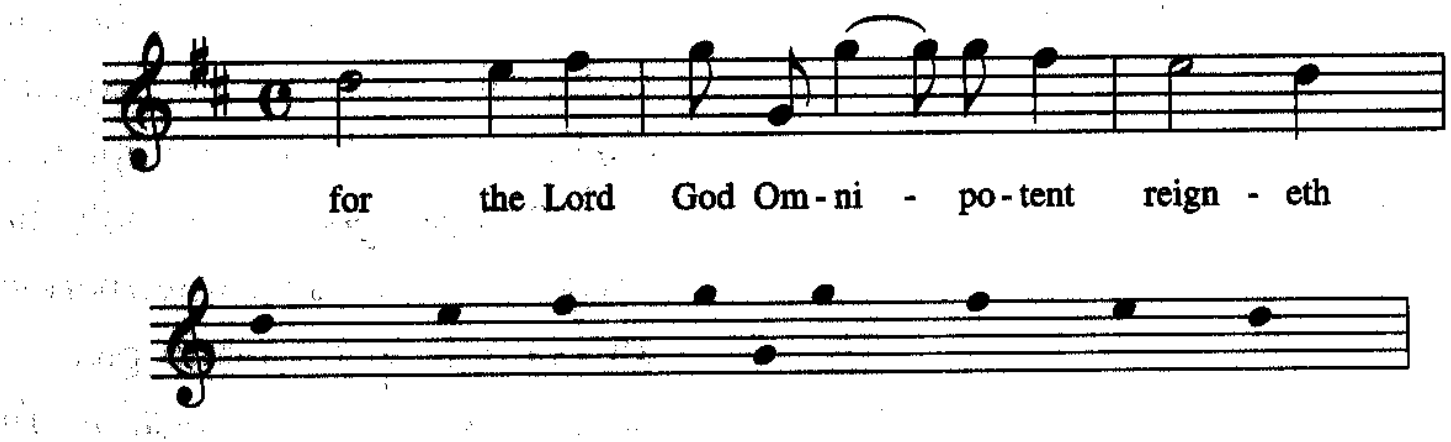
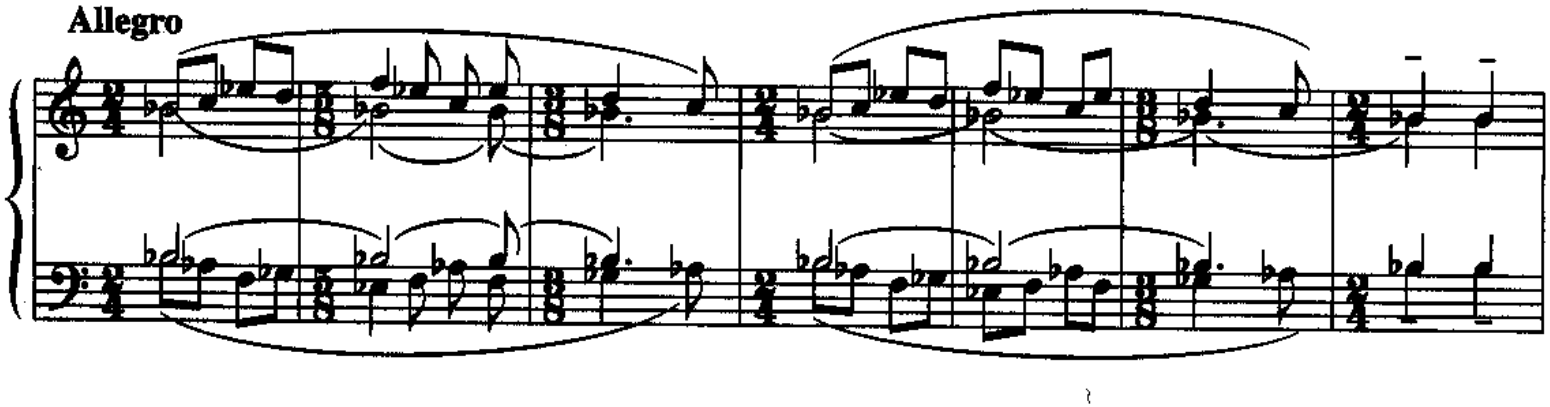


Figure: Johann Sebastian Bach, *A Musical Offering*

Symmetry in Music: Inversion



George F. Handel, *Messiah, Hallelujah chorus* (lose retrograde, form of tone painting)



Béla Bartók, *Mikrokosmos, No. 141, Subject and reflection* (inversion)

Symmetry in Music: Rotation

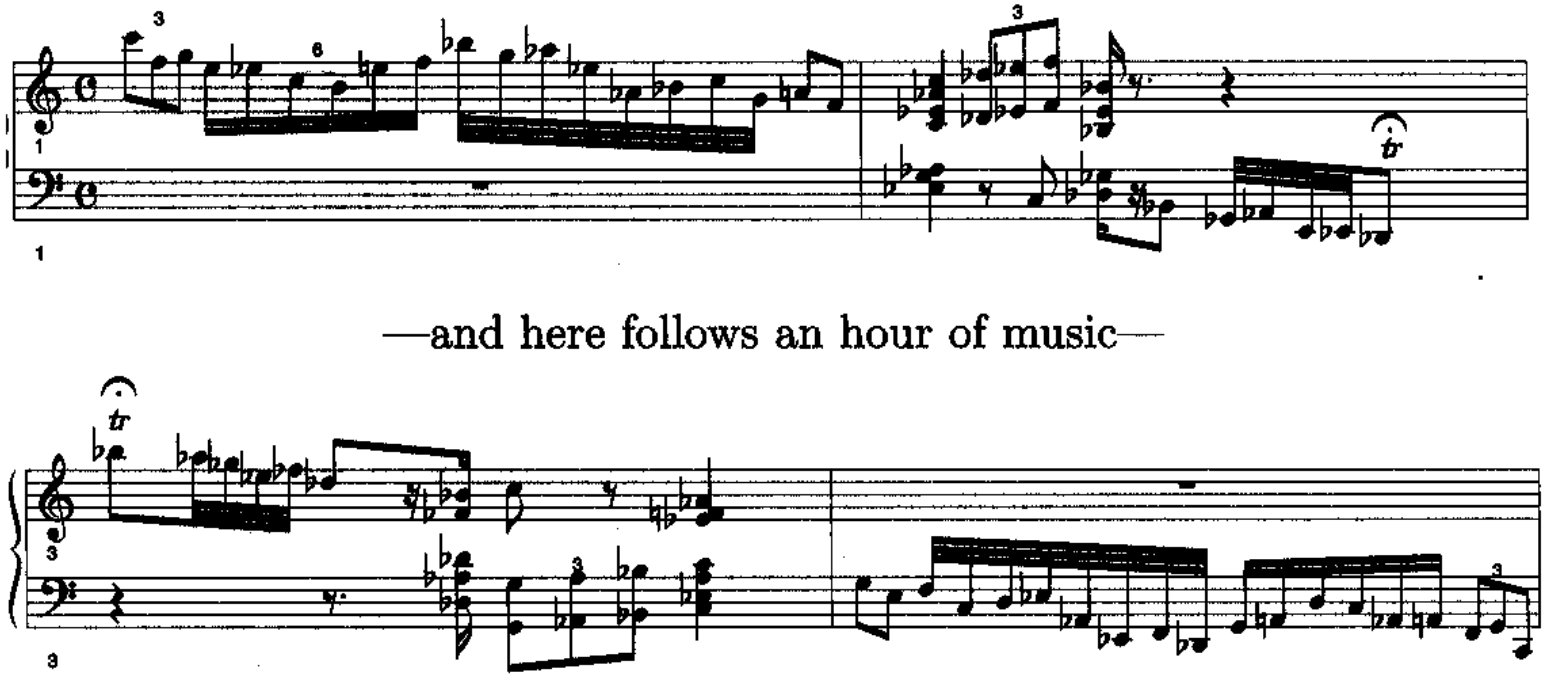
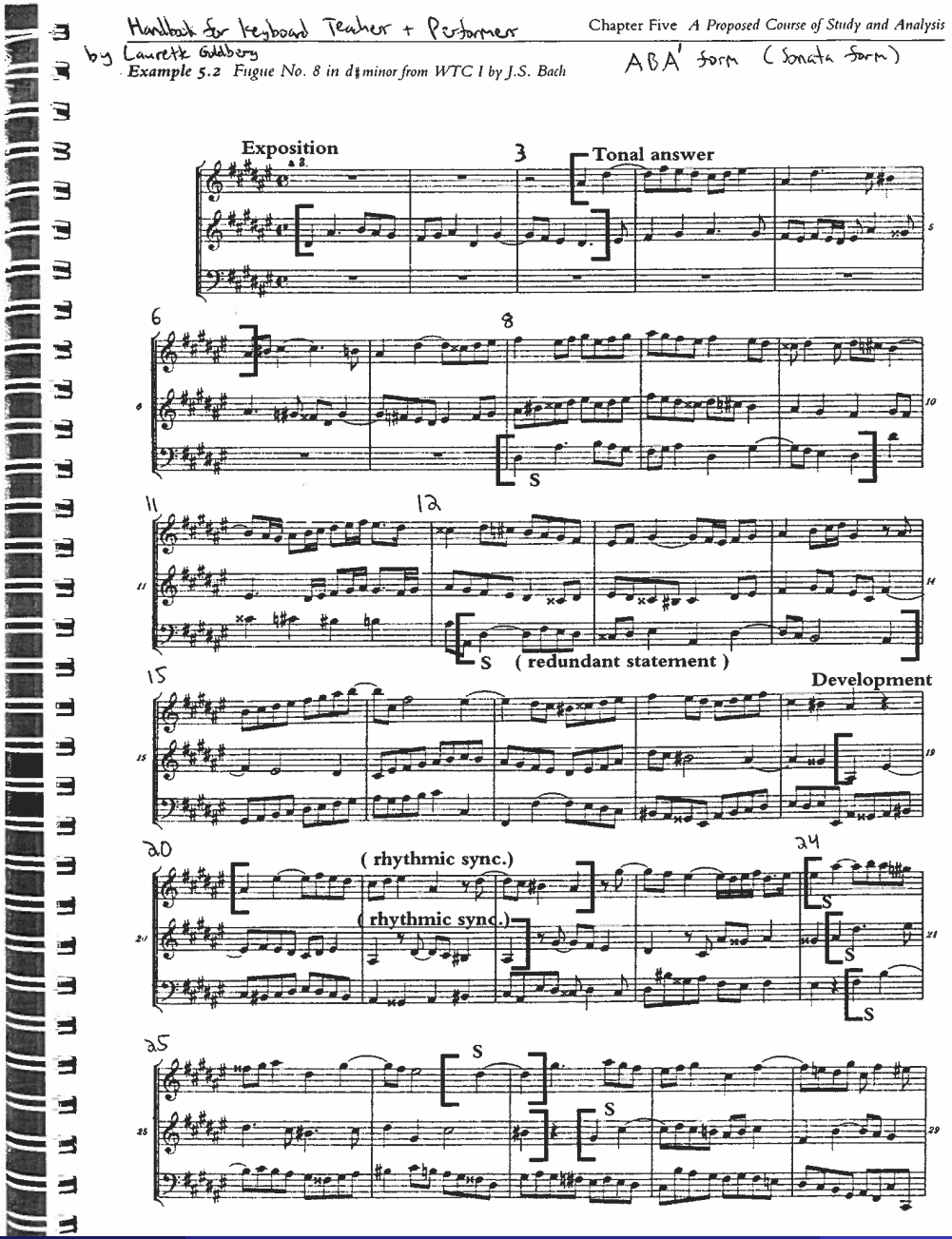
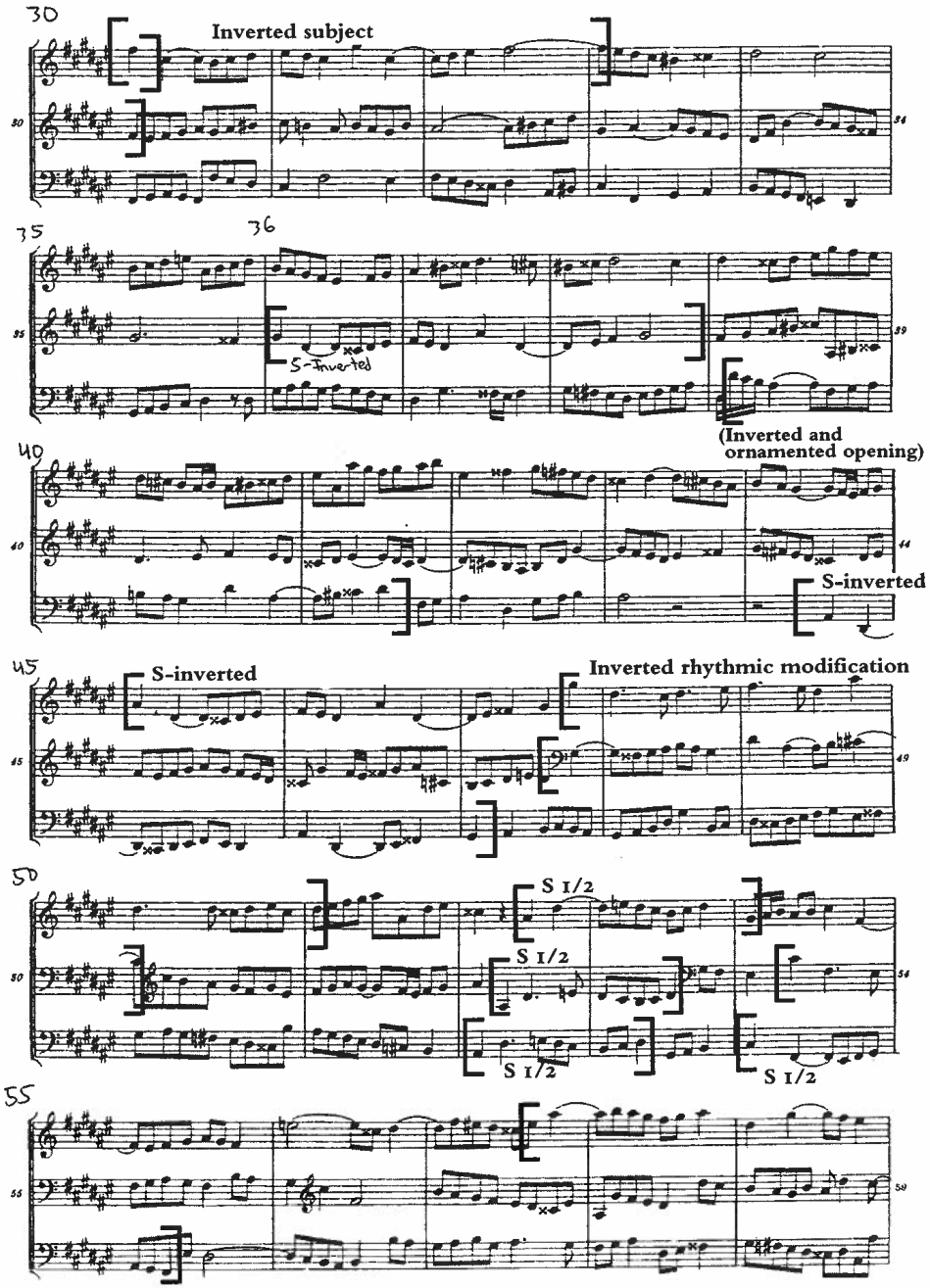
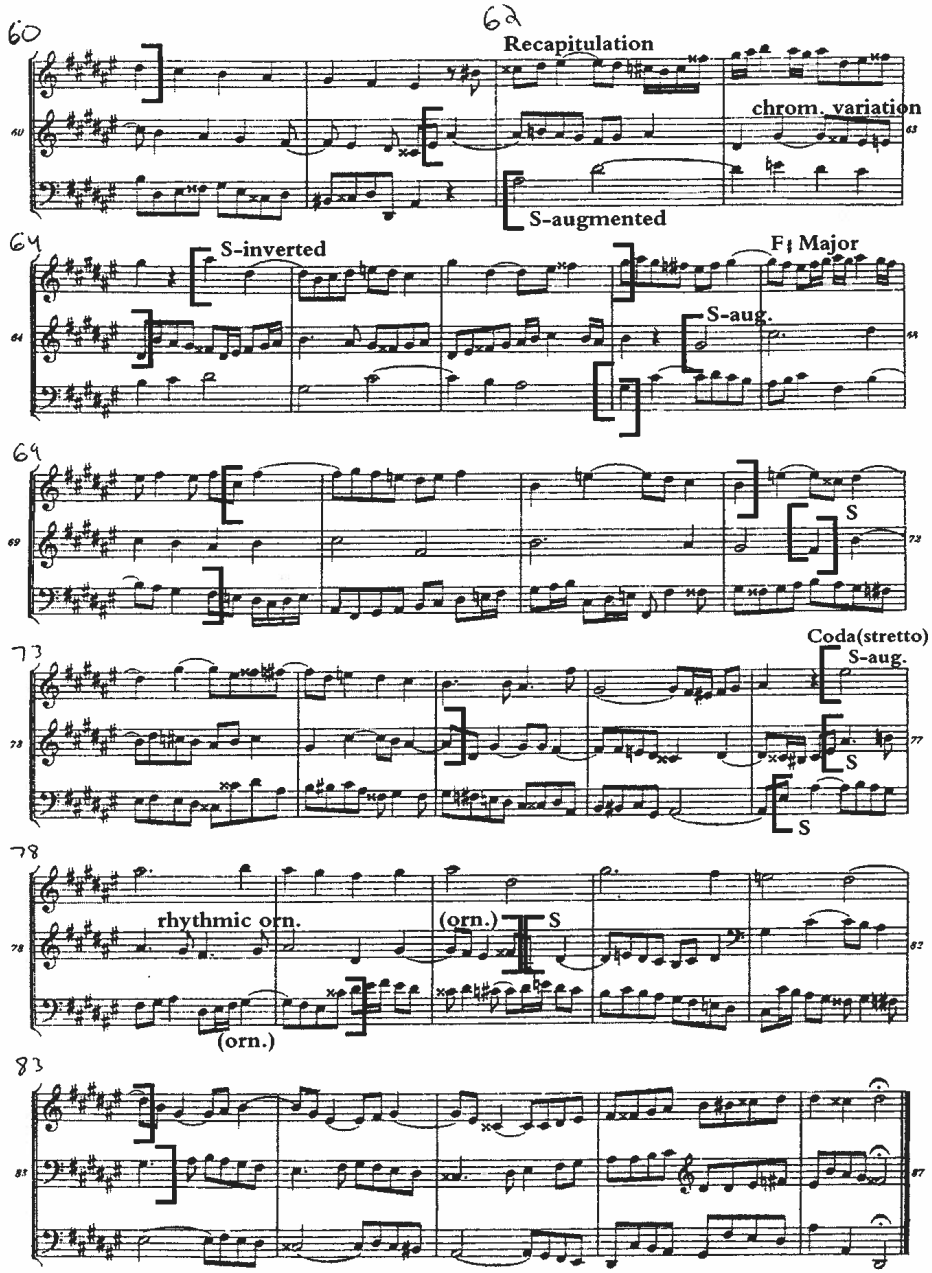


Figure: Paul Hindemith, *Ludus Tonalis* (“Tonal Game”), beginning and end. The ending Postludium is an exact retrograde-inversion of the opening Praeludium.

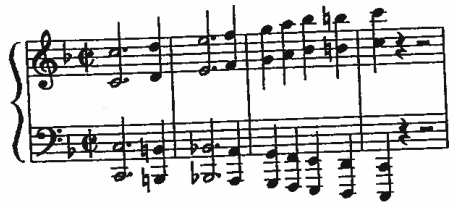
Bach: The Well-Tempered Clavier, Fugue No. 8 in D♯ minor




30
Inverted subject
35
36
S-inverted
(Inverted and ornamented opening)
40
S-inverted
45
S-inverted
Inverted rhythmic modification
50
S 1/2
S 1/2
S 1/2
S 1/2
55


60
62
Recapitulation
chrom. variation
S-augmented
64
S-inverted
F♯ Major
S-aug.
69
S
73
Coda(stretto)
S-aug.
S
S
78
rhythmic orn.
(orn.)
S
(orn.)
83

John Philip Sousa "The Thunderer"



Franz Liszt "Hungarian Rhapsody #2"

THE MATH BEHIND THE MUSIC

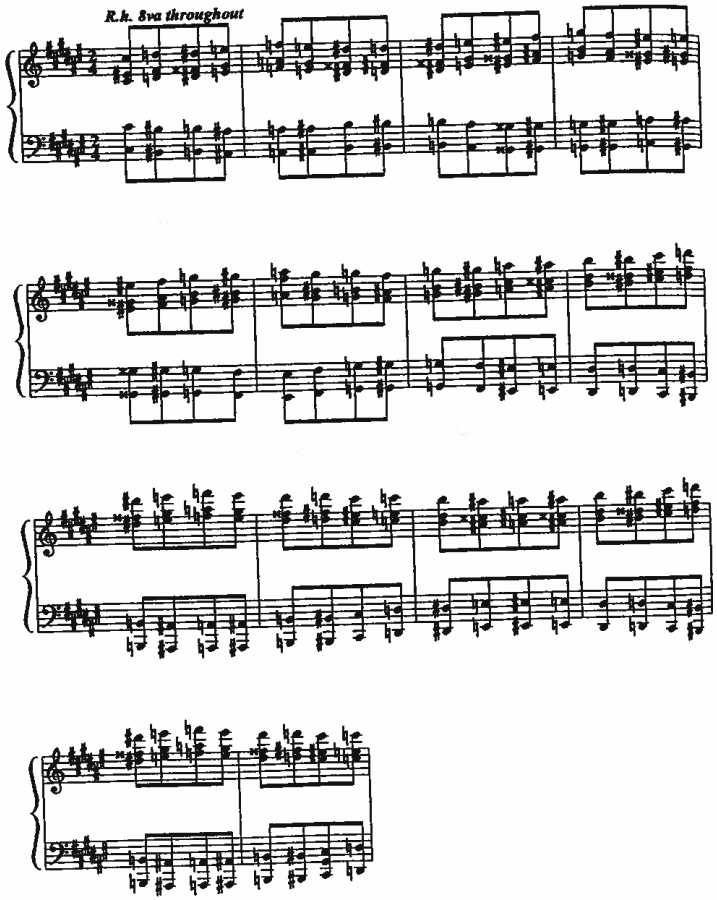


Fig. 11

Combining Symmetries

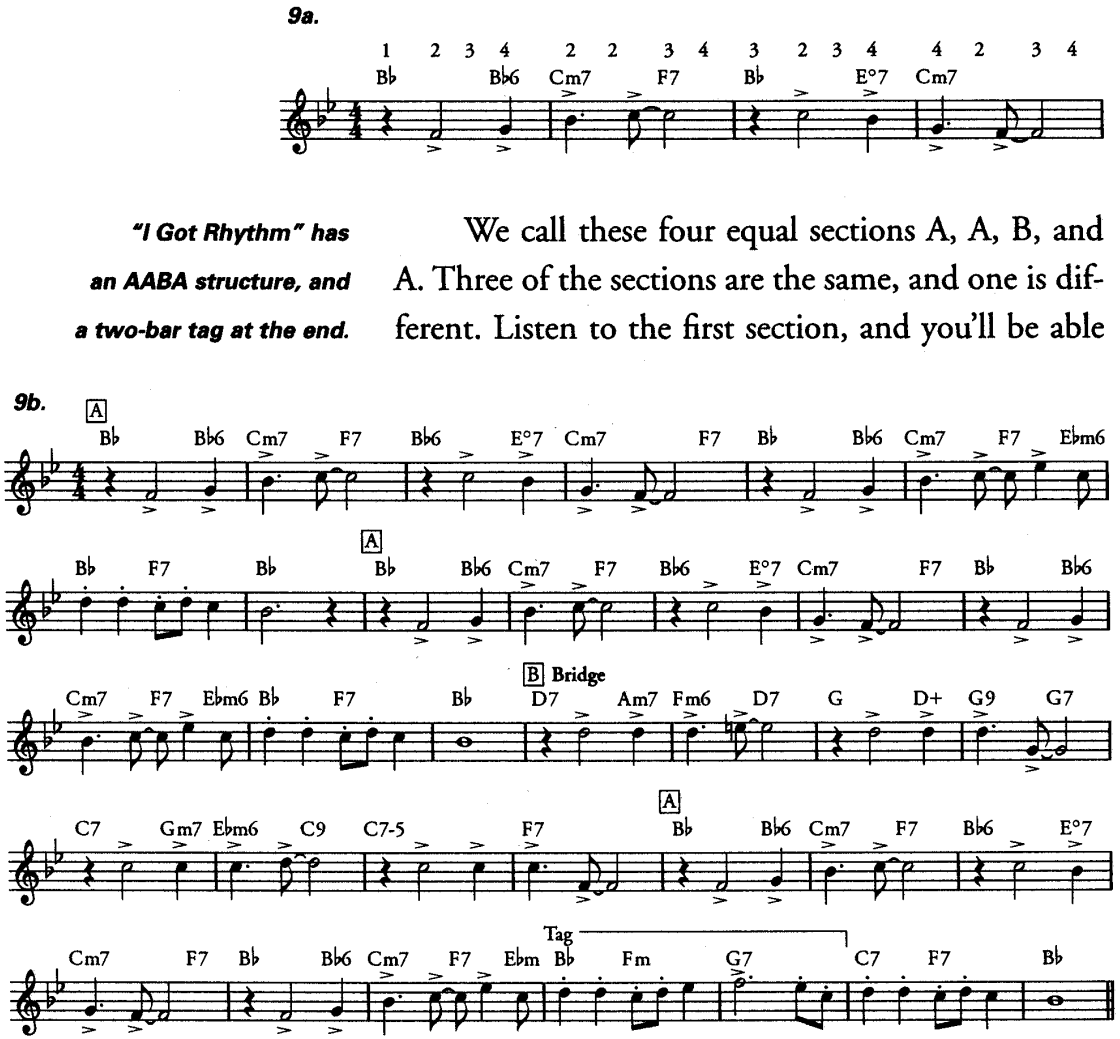
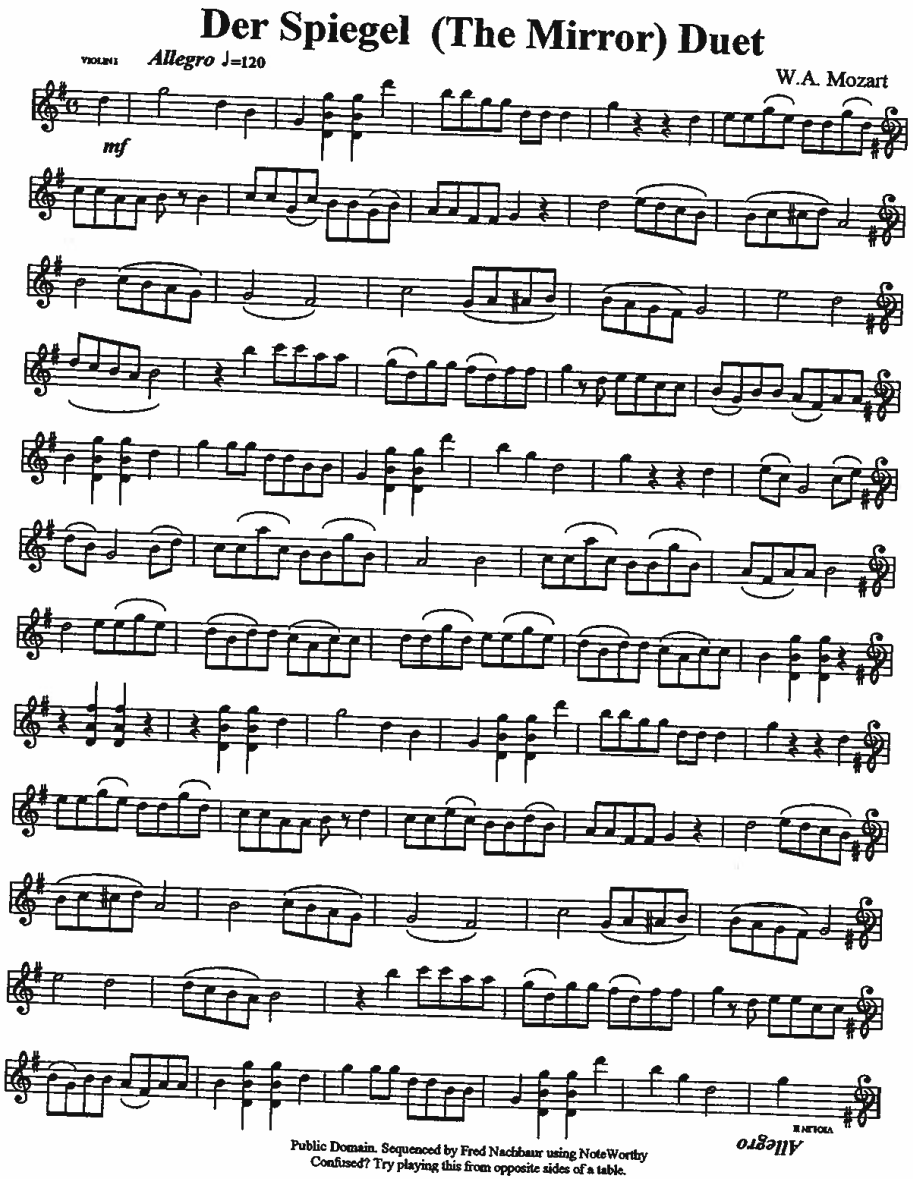


Figure: George Gershwin, *I Got Rhythm*, (transposition, retrograde and inversion, all in one song!)



(Note: the attribution to Mozart is dubious)

Figure 9.6.