

Math/Music: Aesthetic Links

Gareth E. Roberts

Department of Mathematics and Computer Science
College of the Holy Cross
Worcester, MA

Musical Group Theory
Symmetry in Music
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Symmetry in Music: Group Theory

How to get more music out of a little motif:

Translations (shifting graph vertically) $\iff$ Transpositions (shifting notes up or down)

Ex: Ballpark Music

Vertical Reflection (symmetry between right and left) $\iff$
Retrograde (music same forward and backward)

Ex: Lean on Me

Horizontal Reflection (symmetry between top and bottom) $\iff$
Inversion (what goes up, must come down)

Ex: Bach, Bach and more Bach

Symmetry in Music: Retrograde



Figure: Joseph Haydn, *Piano Sonata No. 41, Hob. XVI/26*, Minuet



Symmetry in Music: Retrograde



Figure: Johann Sebastian Bach, *A Musical Offering*

Symmetry in Music: Inversion



for the Lord God Om - ni - po - tent reign - eth

The image shows two staves of music. The top staff contains a melody in G major (one sharp) with a common time signature. The notes are G4, A4, B4, C5, B4, A4, G4, F#4, E4, D4. The bottom staff shows the retrograde inversion of the top staff, with notes G4, F#4, E4, D4, C4, B3, A3, G3, F#3, E3, D3. The lyrics "for the Lord God Om - ni - po - tent reign - eth" are written below the top staff.

George F. Handel, *Messiah*, *Hallelujah chorus* (lose retrograde, form of tone painting)



Allegro

The image shows a piano score for Mikrokosmos No. 141 by Béla Bartók. The tempo is marked "Allegro". The score is in 2/4 time and features a complex, rhythmic melody in the right hand and a more rhythmic accompaniment in the left hand. The melody is characterized by a series of eighth and sixteenth notes, with a prominent use of the interval of a fourth. The score is divided into two main sections by a double bar line, with the second section being a reflection of the first.

Béla Bartók, *Mikrokosmos*, No. 141, *Subject and reflection* (inversion)

Symmetry in Music: Rotation



—and here follows an hour of music—



Figure: Paul Hindemith, *Ludus Tonalis* (“Tonal Game”), beginning and end. The ending Postludium is an **exact** retrograde-inversion of the opening Praeludium.

Bach: The Well-Tempered Clavier, Fugue No. 8 in D minor

Handbook for Keyboard Teacher + Performers Chapter Five: A Proposed Course of Study and Analysis
by Laurette Gabbay Example 5.2: Fugue No. 8 in d minor from WTC I by J.S. Bach ABA' form (Sonata-form)

Exposition 3 Tonal answer

6 8 S

11 12 S (redundant statement)

15 Development

20 (rhythmic sync.) S

24 (rhythmic sync.) S

25 S S

30 **Inverted subject**

35 36

40 (Inverted and ornamented opening)

45 S-inverted Inverted rhythmic modification

50 S 1/2 S 1/2 S 1/2 S 1/2

55

60 **Recapitulation**

chrom. variation

S-augmented

64 S-inverted F# Major S-aug.

69 S

73 Coda(stretto) S-aug. S

79 rhythmic orn. (orn.) S (orn.)

83

John Philip Sousa "The Thunderer"



Franz Liszt "Hungarian Rhapsody #2"

THE MATH BEHIND THE MUSIC

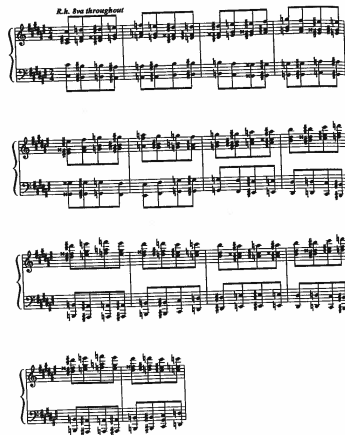


Fig. 11

Combining Symmetries

9a.



**"I Got Rhythm" has
an AABA structure, and
a two-bar tag at the end.**

We call these four equal sections A, A, B, and
A. Three of the sections are the same, and one is dif-
ferent. Listen to the first section, and you'll be able

9b.

Figure: George Gershwin, *I Got Rhythm*, (transposition, retrograde and inversion, all in one song!)

Der Spiegel (The Mirror) Duet

VOCAL *Allegro* $\text{♩} = 120$

W.A. Mozart

mf

Allegro

Public Domain. Synchronized by Fred Nachbauer using NoteWorthy
Codified? Try playing this from opposite sides of a table.

(Note: the attribution to Mozart is dubious)

Figure 9.6.