

Math/Music: Aesthetic Links

CD #2: Modern Mathematical Music: Sets, Rows and Magic Squares

This CD might challenge your concept of “music,” as we include some rather modern “mathematical music” from the 20th century. Featured here are a twelve-tone piano piece by Schoenberg, a composition based entirely on an 8×8 magic square by Davies, an orchestral work using “stochastic music” by Xenakis, and two creative rhythmic works by minimalist composer Steve Reich.

As discussed on CD #2 *Tonality and the Evolution of Polyphony* from last semester, a rather radical shift occurs at the start of the 20th century, when many composers discarded tonality altogether seeking more freedom and flexibility in utilizing all the notes of the chromatic scale, rather than favoring one over the others. Arnold Schoenberg writes, “Tonality is not an eternal law of music, but simply a means toward the achievement of musical form.” Despite this assessment, it is interesting to note that Schoenberg still utilizes the same musical ideas as Bach (inversion, retrograde, transposition, canons) to build his music out of a given tone row. Davies uses the magic square of Mercury to serve as an architectural blueprint for his interesting composition depicting the remarkable properties of light. Xenakis takes the atonality concept even further, composing “sound blocks” based on some very precise mathematical calculations involving probability theory. Finally, Reich has a knack for making slight adjustments rhythmically (shifting or altering accents) to generate whole compositions out of short and simple musical statements.

What to listen for: Portions of the music for the pieces on this CD were distributed in class, although this is hardly the type of music you “follow along with” as you listen. Concentrate as you listen to each piece (don’t give up), trying to discern the composers intent. What emotions and energy do you think the composer is depicting? What strikes you as creative or interesting about the use of instrumentation? How do the mathematical ideas influence the creative process?

1. Arnold Schoenberg, *Suite for Piano, Minuett: Moderato - Trio* Op. 25, 1925. Track 20 on a Deutsche Grammophon CD entitled *Schoenberg: The Piano Music* performed by Maurizio Pollini. This is Schoenberg’s second piece composed for piano during his twelve-tone period. The entire work (not just this movement) is based on the same 12-note tone row. The music for the Trio, which begins about 1:53 into the recording, was distributed in class (see p. 130 of the course text). The opening of the Trio is easily distinguished by its rhythmic punctuations alternating between each hand. The piece is atonal in the sense that there is no central key or tonic around which the music is based. In the Trio Schoenberg uses only the six tone rows P-0, P-6, I-0, I-6, R-6 and RI-6 (see the labels on the music in the text). The number six is significant in that 6 half-steps equals half the octave (the tritone interval) and the tone row begins on an E and ends a tritone away on a Bb. Each of the transformations above (transposition, inversion, retrograde and retrograde-inversion) all consequently begin and end on either E or Bb, making the music flow easier from one row to another. Moreover, there is a canon occurring between the left and right hands, with the rhythm in the right-hand exactly matching that of the left, but coming a full measure (three beats) later.
2. Peter Maxwell Davies, *A Mirror of Whitening Light*, 1977. This live recording of the Manson Ensemble, conducted by Diego Masson, was downloaded off of the composer’s website www.maxopus.com (Sir Davies is widely known as “Max”). The title of the piece refers to the alchemical process

of purification or “whitening” of a base metal into gold. In the opening lecture before the first broadcast performance, Davies describes how the light outside his window workplace in Orkney gave him inspiration for the piece. The composition relies on an 8×8 magic square (the magic square of Mercury) to create the entire set of notes and durations for the piece. Although he claimed in his lecture to have used the magic square of the sun (a 6×6 square), presumably wanting to embody light in some manner, it is clear that Davies actually used the magic square of Mercury instead.

Each number in the square corresponds to a particular note from an 8-note plainchant *Veni sancte spiritus* or its transposed version. The original was transposed to start on each of the eight notes of the chant, thus creating a total of eight 8-note phrases. The magic square and some excerpts detailing its usage were distributed in class (see pp.141 - 143 of the course text). Davies uses patterns from the magic square to assign notes and durations to the different instruments. He describes “these patterns, in the first place, possibly as dance patterns; and one gets to know them by heart. One doesn’t in fact deal with numbers at all. One deals rather as somebody who is dealing with *bell-changes*, with actual patterns with changes” (emphasis added). Concerning the seemingly rigid structure imposed by the magic square, Davies states, “I firmly believe that the more one controls the flow of one’s wildest inspiration, the wilder it sounds. And so when I really wanted to be wild towards the climax of this work, I imposed very rigid rhythmic and tonal controls derived from the plainsong, and from that magic square; and the result is really quite extraordinary I find, even now.”

3. Iannis Xenakis, *Pithoprakta*, 1955 - 1956. Track 4 from a Le Chant Du Monde CD titled Xenakis: Eonta, Metastasis, Pithoprakta featuring the Orchestre National de l’O.R.T.F. under the direction of Maurice Le Roux. A small portion of this music along with the mathematical calculations used to create it were distributed in class. The piece was dedicated to Hermann Scherchen who conducted its premiere in March 1957 in Munich. This innovative work is written for 46 strings (remarkably all playing different parts), 2 trombones, 1 xylophone and 1 wood block. The composers aim was to use probability theory to determine what notes should follow each other and what lengths they should be played, so-called “Stochastic Music,” invented by Xenakis. The piece explores the conflict between continuity and discontinuity by juxtaposing continuous sounds (glissandi in the strings and trombones) with discontinuous ones (pizzicati plucking in the strings, tapping the wood of the strings with the opposite side of the bow, and the sharp, piercing sounds of the wood block.) Xenakis determines the “speed” of a given glissando (the slope found as the ratio of pitch to duration) by using a uniform distribution (sometimes called a normal or Gaussian distribution.) This has the mathematical effect of distributing the speeds equally among all players so that all pitches freely occur along a continuous frequency spectrum. In this way, Xenakis extends Schoenberg’s main goal in 12-tone music of not favoring any one pitch (in this case frequency) over any other. Given Xenakis’ experiences with the Greek Resistance and WWII, it is fairly obvious that this work is the composers musical reflection upon the most tragic of human conflicts – war.
4. Steve Reich, *Clapping Music*, 1972. Track 2 from Disc 3 of a collection of the composers works titled Steve Reich: Works 1965 - 1995 produced by Nonesuch Records. This performance, featuring two players clapping, includes the composer himself and Russell Hartenberger. The music and directions for performance were distributed in class. This clever work consists of 13 measures, each with 6 beats and each repeated 12 times for a total of 156 measures or 936 beats. One performer repeats the exact same rhythmic pattern repeatedly throughout the work (just as the maracas play

steady eighth notes in Reich's *Four Organs* from last semester). The other performer begins in unison with the first player, but then gradually shifts the rhythmic pattern backward (a left shift) by an eighth note at each new bar. Since there are 6 beats or 12 eighth notes, this will produce a new rhythmic pattern in each measure until returning in unison at the 13th measure. Since the original rhythmic pattern is asymmetric, the second performer has a different pattern at each stage of the eighth-note shift. Surprisingly, this simple shift generates entirely new musical ideas throughout the piece. At times the music consists of repeated eighth notes (when the original pattern and its shifted version are completely out of phase), while at other times, the music has a definite dance-like feel due to rests that occur when the original and shifted pattern are more closely aligned.

The underlying mathematical group here is the cyclic group of order 12, the same group that defines pitch classes on the piano or in twelve-tone music. Thus, Reich is rhythmically imitating the identification of the 12 musical pitches of the Western chromatic scale. As is typical with his music, Reich gives very precise instructions to the performers on tempo, duration of the piece (approximately 5 minutes), method of clapping, placement of downbeats, performance style and even electronic amplification. Ironically, Reich does not follow his own instructions, as the first and last measures (the two unison bars) on this recording are repeated only six times. Try and follow the music as each new bar is introduced. It helps to count on half-note pulses: one-two-three, two-two-three, three-two-three, etc. until you reach twelve-two-three, and then a new pattern begins. If you are feeling really confident in your counting skills, try playing this piece with a friend; the challenging role is clapper number two, the one who has to change every twelve bars!

5. Steve Reich, *Six Marimbas*, 1986. Track 3 from Disc 3 of a collection of the composers works titled Steve Reich: Works 1965 - 1995 produced by Nonesuch Records. This performance was recorded by the group *Steve Reich and Musicians* with members of *The Manhattan Marimba Quartet*. The piece is a rescoring for marimbas of Reich's 1973 composition *Six Pianos* and is a more sophisticated development of the rhythmic phase shifting used in *Clapping Music*. By making subtle and gradual rhythmic and melodic changes in the music, a technique often called *minimalist*, the composer creates an almost psychedelic listening experience. The piece opens with four marimbas playing an identical eight-beat rhythmic pattern on different notes in the key of Db major. The other two marimbas then enter together, at first playing only one beat per measure, then two, then three and so on until they are playing the same rhythmic pattern. However, the players shift the pattern at the outset, moving it forward two beats (a right shift). For example, they open by playing the notes from beat 5 of the primary pattern on beat 7, then playing the notes from beat 1 on beat 3, etc. Once this rhythmic canon is complete, other players begin to accentuate interesting melodic features of the original quartet. For example, in measure 18 (roughly 1:43 into the recording), player 6 begins a simple ascending run up and over the top of the scale. This stands out until player 5 begins to emphasize a different motif in measure 22. The dynamics of the piece fluctuate as different players momentarily enter or leave the ensemble. Melodically, the piece is structured into three sections with the first in Db major, the second in Eb dorian and the third in Bb natural minor. It is instructive to watch this piece performed live. A link to a YouTube video recording at the University of Kentucky can be found on the course webpage.