Math/Music: Aesthetic Links
CD #1: Musical Group Theory

This CD contains some of the examples of musical group theory we have discussed in class as well as other illustrative selections. By musical group theory we mean the use of various mathematical symmetries such as translation, horizontal and vertical reflection, and rotation by 180°, which taken in appropriate subsets form a group (e.g., the dihedral group of degree 4 – symmetries of the square). Some composers use these ideas in music quite intentionally to generate new thematic material out of an original idea. Bach was certainly one of the great masters of this, although it is important to note that he did so while never losing site of the musical art in his creations. Not every inversion or retrograde is exactly accurate for want of a musical flavor or style.

What to listen for: The music for many of the pieces on this CD was distributed in class. It is important to follow the music as you listen, as this helps identify the composers intent and reveal the symmetry transformation. It is also a good way to engage your brain while you listen. Try to hear where a retrograde or an inversion takes place in the music. As with CD #2 from last semester, this collection contains a wide variety of musical styles and various tonalities. Try to discern the differences between earlier and later forms of composition as well as between different forms of music (fugue, oratorio, symphony, sonata, song, etc.) The music is mostly organized chronologically with symmetry types underlined and highlighted in boldface for ease of study.

1. Guillaume de Machaut (c. 1300 - 1377), *Ma Fin Est Mon Commencement* (My End is My Beginning). Track 19 from a Hyperion CD entitled Messe De Notre Dame performed by the outstanding Hilliard Ensemble. As the text indicates, this work is a **retrograde** cannon in music and almost in words as well. This is one of the earliest known examples of a retrograde in music that was actually written down. Note also that the piece is sung with tuning using the Pythagorean scale. This is particularly recognizable near the cadences or ends of each phrase. The singers are keeping as many perfect fifths as possible in a 3/2 ratio to produce the Pythagorean tuning using their voices.

2. Johann Sebastian Bach (1685 - 1750), *Fugue, "The Little"*, BWV 578. Track 8 from a CBS Records CD entitled Bach: Great Organ Favorites performed by E. Power Biggs. This is one of Bach's most popular fugues written for the organ. Recall that a fugue is a piece in which a main subject is presented successively in different voices (in this case different registers of the organ), sometimes coming back in inversion or retrograde or even retrograde-inversion. Meanwhile, a countersubject is often presented which complements the main subject heard at the outset. Often this countersubject is more active than the subject and it too may return in inversion or retrograde. Bach was arguably one of the greatest writers of fugues. Try and hear each of the different instances when the main subject reappears, noting that sometimes it shows up in a major key and sometimes in a minor key. The piece is written in G minor although it ends triumphantly on a major chord.

3. Johann Sebastian Bach, *Fugue No. 8 in D♯ minor* from *The Well-Tempered Clavier*, BWV 853, 1722 - 1744. Track 16, Disc 1 from a Radio-Canada CD entitled The Well-Tempered Clavier performed on harpsichord by Scott Ross. Bach wrote 48 preludes and fugues, one each for the 12 major keys and 12 minor keys, over a period of 22 years. The two volume work has been referred to as the pianist's Old Testament. Only through the use of equal temperament tuning

is it possible to play several selections from this masterpiece. The music for Fugue No. 8 in D♯ minor was distributed in class on February 11. As you follow along with the music, notice the ways in which Bach brings the opening subject back, sometimes rhythmically altered (syncopated or augmented), sometimes in **inversion** and sometimes both. Occasionally the subject appears in all three voices, but staggered, as in a round.

4. Johann Sebastian Bach, *The Musical Offering* (excerpt) BWV 1079, 1747. This is track 9 off of the accompanying audio CD to the book *The Math Behind the Music* by Leon Harkleroad, Cambridge University Press, 2006. This is a fun example of a perfect **retrograde** over 18 measures of music. The last nine measures of the bottom part are identical to the first nine measures of the top part played backwards. The same relationship occurs between the last nine measures of the top part and the first nine measures of the bottom part. The music for this excerpt was distributed in class on February 9. Professor John Little will discuss the history of this ingenious work, including other excerpts from it using symmetry, during his special lecture on February 16 at 4:15 pm.

5. George Frideric Handel, *Hallelujah Chorus* from the *Messiah*, 1741. Track 18, Disc 2 from a BMG Entertainment CD entitled Messiah performed by Musica Sacra, directed by Richard Westenburg. Arguably Handel's most famous work, the *Messiah* is a grand oratorio telling the story of Jesus which Handel obsessively composed in only 24 days. The text is a compilation of verses from the Bible. Upon finishing the now famous Hallelujah Chorus, Handel exclaimed "I did think I did see all Heaven before me, and the Great God Himself!" Early on in the chorus (about 22 seconds in) we hear all the voices in unison singing "for the Lord God Omnipotent reigneth," a phrase which if one neglects duration, is in **retrograde** (see the top of p. 103 in the course text.) By using vertical symmetry, Handel is announcing the brilliance and balance of The Lord God (perhaps an early form of tone painting.) To emphasize this significance, the motif appears throughout this movement. Traditionally the audience stands when the Hallelujah Chorus is sung during a performance of the Messiah, often a powerfully spiritual moment for performers and audience alike.

6. Joseph Haydn, *Piano Sonata No. 41, "Minuet and Trio"* in A major, Hob. XVI:26, 1774-1775. Track 13, Disc 6 from the complete collection of Haydn's Piano Sonatas performed by John McCabe (Decca Record Company Limited, London). The opening minuet is titled "Menuet Al Rovescio" (Minuet in Reverse) and is an exact musical palindrome (**retrograde**) with the vertical reflection occurring at the end of measure 10. This music was distributed on February 9 and can be found on page 90 of the course text. The $\frac{3}{4}$ meter seems crucial to making the retrograde work harmonically as Haydn often emphasizes two of the three notes of the underlying chord in beats one and three of each measure. Thus, on playing such a measure backwards, the chord heard remains unchanged. Haydn was so proud of his clever musical palindrome that he re-worked it into his Sonata No. 4 for Piano and Violin as well as his Symphony No. 47.

7. Ludwig Van Beethoven, *Symphony No. 5 in C minor*, 1st movement, Op. 67, 1807-1808. Track 1, Disc 3 from the complete collection of Beethoven's Nine Symphonies (he actually wrote 10, but the 10th was unfinished) performed by the Chicago Symphony Orchestra under the direction of Sir Georg Solti. This famous symphony took the idea of elaborating upon a simple motif to new heights. The opening "da da da dum" motif, once called "fate knocking at the door" by Beethoven himself, is **transposed**, inverted, elongated, shortened and continually repeated throughout the work to create the bulk of the music. The entire first minute of this symphony consists solely of

this famous motif. Although only the first movement is included here, you are encouraged to listen to the entire symphony (multiple versions available in the music library) to see how Beethoven brings the motif back unexpectedly in the final movement.

8. John Philip Sousa, *The Thunderer* (excerpt), 1889. Track 11 off the Harkleroad CD, the opening to this march is an easily discernible **inversion**. Note that this inversion is different than those of Bach in that it happens *simultaneously* rather than delayed. It is not exact however, staying in the same key of F in each hand. If one follows the names of the notes, it is also a retrograde between the top and bottom lines. The music for this brief excerpt was distributed in class on February 11.

9. Franz Liszt, *Hungarian Rhapsody #2* (excerpt), 1847. Track 13 off the Harkleroad CD, this is a fine example of combining many of our symmetry transformations. Here we can find **transpositions**, **inversions** and **retrograde** in just a few measures. Like the Sousa excerpt before, the symmetry relationships are fairly easy to hear and see. The full piece is one of the most popular piano solos around and was used in many animated cartoons including the Bugs Bunny short *Rhapsody Rabbit* (Warner Brothers) and Tom and Jerry's *The Cat Concerto* (MGM) which won the 1946 Academy Award for best cartoon. The music for this excerpt was distributed in class on February 11.

10. Béla Bartók, *Mikrokosmos*, Book 6, BB 105: No. 141 *Subject and reflection*, 1926 - 1939. Track 45 from a complete recording of this famous set of piano pieces, performed by Jenö Jandó (Naxos). The entire Mikrokosmos consists of 153 pieces for piano ranging in difficulty from beginner to advanced and featuring some interesting musical traits including the use of whole-tone scales, Hungarian folk tunes, syncopation and modal harmonies. In Bartók's own words, Mikrokosmos "appears as a synthesis of all the musical and technical problems which were treated and in some case only partially solved in the previous piano works." This particular piece is a fine example of an **inversion** and as with the previous two pieces, this inversion happens simultaneously (at least for the opening half) so that the musical symmetry is readily apparent in the score as well as upon listening. The inversion is exact in the sense that the musical intervals are mirrored precisely (right-hand up a minor third means left-hand down a minor third), a compositional technique not harmonically accepted during Bach's time. The inversion appears to be syncopated in the second half of the piece. The music for the first 7 measures was distributed in class on February 9.

11. George Gershwin, *I Got Rhythm*, 1930. Track 6 off of a Concord Records, Inc. CD entitled Someone to Watch Over Me sung by Susannah McCorkle. This is a quaint intro track discussing the lyrics of the tune written by the composer's brother, lyricist Ira Gershwin.

12. George Gershwin, *I Got Rhythm*, 1930. Track 7 off of the previous CD. Gershwin, who grew up in Brooklyn, NY, the son of Russian-Jewish parents, wrote this popular tune in his musical *Girl Crazy*. The music for this song was distributed in class on February 11. The main opening melody is 4 bars with a simple **retrograde** at the end of bar 2 (with an important shift in the rhythm to give it a jazzy feel.) The form of the song is an AABA structure with the B section featuring an **inversion** (roughly) as well as a **transposition**. Thus, as with the piece by Liszt, this simple 32 bar song contains three of our four mathematical symmetries.