

# Math/Music: Aesthetic Links

## Homework Assignment #1

**DUE DATE: Monday, February 21, start of class.**

Homework should be turned in at the beginning of class. You should write up solutions neatly to all problems, making sure to show all your work. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your own work. Please cite any references (web based or text) that you may have used for assistance with the assignment.

**Note:** Please list the names of any students or faculty who you worked with on the assignment.

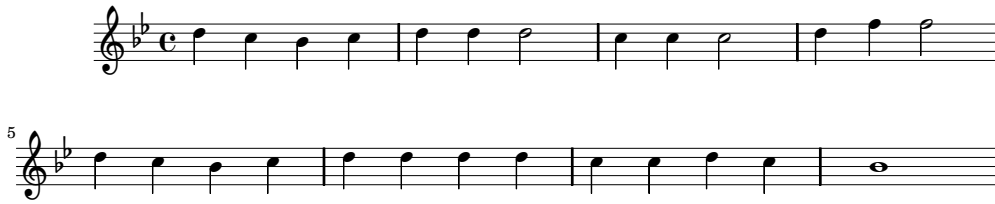
Limerick - kciremiL

*First, let me explain that I'm cursed;  
I'm a poet whose time gets reversed.  
Reversed gets time  
Whose poet a I'm;  
Cursed I'm that explain me let, first.*

1. Listen to CD #1: *Musical Group Theory* available on the Montserrat Moodle page for the course. Liner notes are available from the course webpage in the Handouts section and much of the written music for the CD was distributed in class on Feb. 9th and 11th. You may be tested on some of this music so be sure to read the liner notes and listen carefully. One approach to effectively absorbing the music is to make brief notes about each piece, listing significant details as you listen.
2. Read Chapter 6 of the course text, *The geometry of music*, by Wilfrid Hodges.
  - a. On page 99, Hodges gives five different symmetry classes,  $p1$ ,  $ph$ ,  $pv$ ,  $p2$ ,  $phv$ . For each of the letters **B**, **A**, **L**, **T** and **Θ**, give the corresponding symmetry class.
  - b. Give two examples of music using symmetry  $p2$ .
  - c. What type of symmetry does Mozart use in his *Clarinet quintet, K381*?
3. In the key of  $B\flat$ , write out the first eight measures of *Mary Had a Little Lamb* after applying an inversion about the note  $B\flat$ , staying in the *same key*. Do this in the treble clef with the correct key signature. Music is provided on the top of the next page. (You do not need to include or invert the text.)
4. In the key of  $B\flat$ , write out the first eight measures of *Mary Had a Little Lamb* after applying a retrograde-inversion about the note  $B\flat$ , staying in the *same key*. Do this in the treble clef with the correct key signature.
5. Play the original and each of the two transformed melodies on the piano. How do the new melodies sound? Which do you prefer?

## Mary Had a Little Lamb

music by Lowell Mason



6. There is an important distinction to be made when using a musical inversion. In general, a strict inversion applied mathematically as an exact horizontal reflection will *not* stay in the same key. Composers get to decide whether they want to leave the key or not. In class, we played a descending C major scale inverted *exactly* about the tonic and obtained a scale that was in the key of  $A\flat$  major. The notes (descending), using diatonic scale spellings, were C,  $B\flat$ ,  $A\flat$ , G, F,  $E\flat$ ,  $D\flat$ , C which are the same notes in an  $A\flat$  major scale, but beginning on the 3rd note of the scale.
- If you invert a G major ascending scale about the starting note G, what key is the resulting scale in?
  - Going in the other direction, what major scale, when inverted exactly about its tonic, gives precisely the notes in the C major scale?
  - Bonus:** State a general rule for finding the new key when inverting (exactly) a major scale about its tonic. How do you find the new key quickly?
7. **Closure:**
- Which of the following sets are closed under addition? Explain.  
(i) The integers    (ii) The rational numbers    (iii) The irrational numbers
  - Which of the following sets are closed under multiplication? Explain.  
(i) The integers    (ii) The rational numbers    (iii) The irrational numbers
8. Finish completing the group multiplication table (started in class) for the symmetries of the square. This set is called the **Dihedral group of degree 4**, denoted  $D_4$ .
9. Notice that the completed multiplication table shows that the symmetries of the square are closed under composition. In other words, composing two symmetry operations of the square yields one of the eight symmetries. Associativity follows from the definition of composition of functions and the identity element is contained in the set of symmetries. List the inverse of each of the eight elements of  $D_4$  and conclude that  $D_4$  is a group of order 8.
10. What do you notice about any row or any column in the multiplication table?
11. Is the group  $D_4$  commutative? In other words, is it true that  $a * b = b * a$  for **every** element  $a, b$  in the group  $D_4$ ? Explain.
12. Using your table, in general, what is the composition of two rotations? of two reflections? of a rotation and a reflection?