Mathematics and Music: Aesthetic Links

MONT 111N-01, MWF 9:00 - 9:50, Brooks Center 452, Spring 2011

Professor Gareth Roberts

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Office hours: Mon. 11:00 - 12:00, Tues. 1:30 - 3:00, Wed. 8:00 - 8:50 (in Wheeler), Fri. 11:00 - 12:00 or by appointment.

Required Texts: *Music and Mathematics: From Pythagoras to Fractals*, edited by Fauvel, Flood and Wilson, and *Collapse* by Jared Diamond.

Webpage: `http://mathcs.holycross.edu/~groberts/Courses/Mont2/homepage.html` Homework assignments, handouts, schedule changes, exam materials, useful links and other important information will be posted at this site. Please bookmark it!

Course Content: At first glance, mathematics is often considered a "hard" science while music is deemed a staple of the humanities. However, in addition to the many structural traits shared by each field (discussed last semester), there are also many aesthetic and artistic links between the two disciplines. For example, both fields have produced great child prodigies (for example, Mozart and Gauss). Parents play Mozart and Bach to their children, even in the womb, to help foster their brain development and analytic skills. Many mathematicians are outstanding musicians, while many musicians, in particular composers, possess sharp mathematical minds. Scholars often speak of the "beauty" and "purity" of mathematics, although the same lofty descriptions could equally apply to music. While music has the obvious capacity to move the spirit, great mathematical discoveries and insights are often accompanied with an overwhelming sense of elation. The great mathematician Andrew Wiles wept on camera while speaking about his incredible proof of Fermat's Last Theorem.

Composers and musicians, whether they are cognizant of the fact or not, use mathematical concepts in their creations. Bach frequently used mathematical operations such as translations and reflections in his fugues to create wonderfully rich counterpoint. Bell ringers in towers throughout England have been using permutations, symmetry and group theory in change ringing (sometimes called campanology) to announce important events since the early 1600's. Music theorists and mathematicians alike have claimed the appearance of the golden section in compositions of some of the great composers (Mozart, Debussy and Bartók, to name a few). The modern composer Xenakis used computers and probability theory to create his "stochastic music."

This course will explore the myriad of ways composers have used mathematical thinking to create meaningful music. We will consider many examples, employing mathematical concepts and techniques to gain a deeper understanding of the music.

A tentative outline of the course is given below. Although we will cover material from many chapters of the primary course text, specifically Chapters 6, 7, 8 and 10, there will also be some handouts used for certain topics. One class will be set aside to learn about software for writing and hearing a musical composition.

- Introduction: course overview (1 class)

- Book Discussion: *Collapse* (4 classes)
- Musical Group Theory: symmetry, group theory, symmetries of the square, examples (4 classes)
- Guest Lecture: "Symmetry in Music," Professor John Little, Feb. 16, 4:15 pm
- Change (Bell) Ringing: rules of an extent, permutations, more group theory (4 classes)
- Modern Music: Schoenberg (twelve-tone method), Davies, Xenakis, Reich (5 classes)
- Midterm Exam (in class)
- The Golden Section: Fibonacci numbers, nature, Mozart, Bartók (5 classes)
- Fractals and Chaos: iteration, the butterfly effect, Ligeti, Johnson (4 classes)
- The Mozart Effect: music's ability for brain development and healing (3 classes)
- Final Project preparation (3 classes)
- Final Project presentations (last 3 classes)

Other Important Dates:
- Lecture on Medical Ethics: Thursday, Feb. 17
- Evening Concert: Boulez, Xenakis, Bach, Thursday, Feb. 17
- Natural World Cluster: Field Trip to Boston Museum of Science, Saturday, April 16
- Natural World Cluster: End of the Year Banquet, Monday (evening), May 2

Course Objectives:
1. Investigate the multiple connections between mathematics and music.
2. Develop skills in critical thinking and abstract reasoning.
3. Develop a deeper appreciation for music.
4. Integrate your artistic and analytical skills.
5. Participate in and contribute to the common events, lectures, etc. of the Natural World Cluster of the *Montserrat* program.

Homework: There will be approximately 4 homework assignments for the semester. The smaller number is to provide you with ample time to write a research paper and complete your final project. Assignments will be posted on the course web page. While you are allowed and encouraged to work on homework exercises with your classmates, the solutions you turn in to be graded should be your own. Take care to write up solutions **in your own words**. Plagiarism will not be tolerated and will be treated as a violation of the Departmental Policy on Academic Integrity.

Concert Reviews: You are required to attend two musical performances during the semester and turn in a typed, 1-2 page review of each concert. The purpose of these reviews is to enhance your musical appreciation, to support your fellow students and the arts, and to notice and describe any possible connections to course material. Your review should include basic information about the concert (location, date, performers, pieces, composers, etc.), your opinion of the concert (strengths and weaknesses) and some connection to current or past (last semester) course material. A schedule of upcoming concerts is linked from the course homepage.

Midterm Exam: There will be one midterm exam given during class on Friday, March 25. We will review for the midterm during the class period before the exam. If you have any specific learning disabilities or special needs and require accommodations, please let me know early in the semester so that your learning needs may be appropriately met. You will need to contact the director of Disability Services in Hogan 215 (x3693) to obtain documentation of your disability.

Research Paper: There will be a research paper due on Wednesday, April 20, in which you will investigate one composer and a particular piece (or set of pieces) written by that composer demonstrating some mathematical concept or technique. Your paper will feature an historical element as well as an investigative component analyzing the work(s) in question.

Final Project: You will be expected to complete a final project consisting of a musical composition and performance demonstrating some of the mathematical concepts learned in the course. A brief, typed report should accompany your composition explaining the mathematical connections and rationale in your work. You will also give a brief presentation describing the work immediately before its performance. The performances will take place during the last three class periods of the semester. A few class periods will be allocated for feedback and constructive criticism before the final performances.

Academic Integrity: The Department of Mathematics and Computer Science has drafted a policy on academic integrity to precisely state our expectations of both students and faculty with regards to cheating, plagiarism, academic honesty, etc. You are required to read this policy and sign a pledge agreeing to uphold it. A violation of the Departmental Policy on Academic Integrity will result in a 0 for that assignment (or exam) and a letter describing the occurrence of academic dishonesty will be sent to your Class Dean.

Grade: Your course grade will be based on the following breakdown:

- classroom participation/interest 5%
- concert reviews 5%
- *Collapse* paper 10%
- homework assignments 15%
- research paper 15%
- midterm exam 20%
- final project 30%

How to do well in this course:

- Attend class, participate and ask questions.
- Do your homework regularly.
- Work with your classmates.
- Ask for HELP when necessary.

> *Mathematics and music, the most sharply contrasted fields of scientific activity which can be found, and yet related, supporting each other, as if to show forth the secret connection which ties together all the activities of our mind, and which leads us to surmise that the manifestations of the artist's genius are but the unconscious expressions of a mysteriously acting rationality.*
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> Hermann von Helmholtz, 1884