Math/Music: Structure and Form

Homework Assignment #5 (Last One!)

DUE DATE: Fri., Dec. 9, start of class

This homework is a group assignment, where you are allowed (and encouraged) to work with your classmates to complete the assignment. Groups should contain between one and three members, and turn in **one** set of solutions for the entire group. Each student in the group will receive the same grade.

As always, you should write up solutions neatly to all problems, making sure to show all your work. Please cite any references (web based or text) that you may have used for assistance with the assignment.

Math is the only place where truth and beauty mean the same thing.

– Danica McKellar (former star of the *The Wonder Years* and author of the bestseller *Math Doesn't Suck*)

1. Read Chapter 4 of the course test, *Faggot's fretful fiasco*, by Ian Stewart.

2. What was the specific mistake that Jacob Faggot made which led him to dismiss Strähle's construction as inaccurate? Derive (not just cite, but derive) the correct value Faggot should have obtained. There is a small error in the text here (discovered by some of my previous students). Can you find it?

3. Compute the continued fraction expansion for the rational number $\alpha = 53/14$. Be sure to show your work. Give your answer in the form $[a_0; a_1, a_2, \ldots]$.

4. Compute the continued fraction expansion for the irrational number $\alpha = \sqrt{6}$. Be sure to show your work. Give your answer in the form $[a_0; a_1, a_2, \ldots]$. *Hint:* Your answer should be periodic.

   Extra Credit: Prove that your expansion does indeed equal $\sqrt{6}$ by using the method of self-similarity, as demonstrated for $\sqrt{2}$ on the class worksheet on continued fractions.

5. Consider the irrational number α with periodic continued fraction expansion $[1; 1, 1, 1, \ldots]$.

   a) Compute the first seven convergents of α, that is, compute p_n/q_n for $n = 0, 1, \ldots, 6$. The numbers in the numerator and denominator are famous. Where do they come from?

   b) Find the exact value of α by using the method of self-similarity, as demonstrated for $\sqrt{2}$ on the class worksheet on continued fractions. What is the name of the number α?

6. Recall the importance of the number $\log_2(3/2)$ when attempting to find good approximations to a true or just perfect fifth in an equally tempered tuning system (one that uses the same half-step throughout to build the entire scale).

a) Compute the first seven terms of the continued fraction expansion of $\log_2(3/2)$, that is, compute a_n for $n = 0, 1, \ldots, 6$.

b) Using the recursion formulas, compute the first seven convergents of $\log_2(3/2)$, that is, compute p_n/q_n for $n = 0, 1, \ldots, 6$. Be sure to show your work.

c) If you have done part **b)** correctly, you should recognize the fraction obtained for $n = 4$. Explain how this relates to Equal Temperament.

d) Why would it "work" to divide the octave into 53 equal parts, creating a scale with 53 notes equally spaced by the half step $H = 2^{1/53}$? In this case, what would the value of the frequency multiplier be to raise the pitch a perfect fifth? (This is the number to multiply the tonic frequency by in order to raise the pitch a P5.) How many cents is this ratio (to three decimal places) and how close is it (in cents, three decimal places) to a just perfect fifth?

e) Assuming you created a 53 note scale with equally spaced half steps, how many half steps would be in a major third? In this case, how many cents is the frequency multiplier used to raise the pitch a major third (to three decimal places)? How close is it (in cents, three decimal places) to a just major third? Would you need to raise or lower the pitch in your new scale to obtain a just major third?