# Math/Music: Structure and Form

# Homework Assignment #4

## DUE DATE: Mon., Nov. 28, start of class.

You should write up solutions neatly to all problems, making sure to show all your work. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your own work. Please cite any references (web based or text) that you may have used for assistance with the assignment.

**Note:** Please list the names of any students or faculty who you worked with on the assignment.

> *Equal temperament is a lot easier to use than Just Intonation, but I find it lacks expressiveness. It sounds dead and lifeless to me. As soon as I began working microtonally, I felt like I moved from black and white into color. I found that certain combinations of intervals moved me in a deep physical way. Everything became clearer for me, more visceral and expressive. The trade-off is that I had to be a lot more careful with my compositions, for while I had many more interesting consonant intervals to chose from, I also had new kinds of dissonances to avoid.* Robert Rich (composer)

1. Read Chapter 1 of the course text, *Tuning and temperament: closing the spiral* by Neil Bibby. As you read, it is useful to do some of the computations in the text to further your understanding. Much of this material will be covered in class.

2. What famous scientist tried to solve the musical difficulties of Just Intonation by using symmetry and linking the seven notes of the major scale to the seven colors of the spectrum?

3. Consider the C major Pythagorean scale. By ascending and descending by the correct number of perfect fifths and octaves, derive the frequency ratios for F♯ and G♭ given in the text on page 19, assuming that middle C is the tonic (Do). For F♯, you should start on middle C and go up in pitch by perfect fifths, then down by the correct number of octaves to get back to the F♯ just above middle C. For G♭, you should start on middle C and go down in pitch by perfect fifths, then up by the correct number of octaves to get back to the G♭ just above middle C. The circle of fifths is helpful here. What is the ratio of the two fractions F♯ : G♭? This should look familiar.

4. Recall that the **overtone series** of a frequency $f$ consists of the sequence of higher and higher multiples of $f$: $f, 2f, 3f, 4f, 5f, 6f, 7f, 8f, \ldots$

   **a.** List the first twelve frequencies (including the fundamental) in the overtone series for 300 Hz.

   **b.** List the first twelve frequencies (including the fundamental) in the overtone series for 400 Hz.

   **c.** Circle the frequencies that the overtone series for 300 Hz and 400 Hz have in common. What is the musical interval between these notes?

   **d.** Form a concise statement concerning the overlap of two overtone series with frequencies $3f$ and $4f$.

5. Beginning on the note an octave below middle C, use music staff paper to write the first ten notes corresponding to the frequencies in the overtone series. Start in the bass clef. Give the musical interval between successive notes in the series.

6. In a brief essay, compare and contrast the three tuning systems: **Pythagorean Tuning**, **Just Intonation** and **Equal Temperament**. What are the strengths and weaknesses of each system? Which intervals are the same and which are different? What does Bibby mean by "closing the spiral?" Your class notes should be helpful here.

7. Make a chart showing the frequencies (rounded to the nearest tenth, i.e., one decimal place) of all the notes in the A major scale starting on A 220 Hz. Do this for **each** of the three different tuning systems: Pythagorean Tuning, Just Intonation and Equal Temperament.

8. Assuming that A 440 Hz is the note A above middle C, find the following frequencies in Hz (round to the nearest tenth) using the given tuning system. Be sure to explain how you arrived at each answer. *Hint:* If going up by a certain musical interval means multiply by the ratio $r$, then going down by that same interval means dividing by $r$.

   **a.** F above middle C using Just Intonation

   **b.** F above middle C using Pythagorean Tuning

   **c.** Middle C using Just Intonation

   **d.** Middle C using Equal Temperament

9. Using the frequency ratios of the Pythagorean Scale, what ratio do you multiply a frequency by to raise the note up a minor third? Due to the Pythagorean comma, this value is not unique. Therefore, find the ratio with the smallest possible numerator and denominator. For example, going up by three half-steps yields a ratio with very large (too large) numbers in the numerator and denominator.

10. Using Equal Temperament, what factor do you multiply a frequency by to raise the note up a tritone? What factor raises the note by a minor seventh? Be sure to simplify your answers.

    **Recall:** A number is **rational** if it can be expressed as the ratio of two integers $\frac{p}{q}$. A number is **irrational** if it is not rational.

11. Using a proof technique demonstrated in class, prove that $\sqrt{3}$ is irrational and prove that $2^{1/12}$ is irrational.

12. Prove that the sum of any two rational numbers is rational. Mathematically, this means that the set of rational numbers is *closed* under addition. *Hint:* How do we add two fractions together? Show that the result is still rational.

13. Prove that the sum of a rational and an irrational number is always irrational. *Hint:* Suppose this wasn't true and that the sum was rational. Why does this lead to a contradiction of the fact proven in the previous question?

14. Are the irrational numbers closed under addition? In other words, is it true that the sum of two irrationals is **always** irrational? Explain with a proof or provide a counterexample.