MONT 108N   Exam 1 SOLUTIONS
Math/Music: Structure and Form

October 21, 2011   Prof. G. Roberts

1. **Listening Questions:** Choose the best answer available. (5 pts. each)

   (a) An excerpt of a piece of music will be played. Which of the following best describes the work heard?

   (i) The piece is in $\frac{2}{4}$ time invoking a lively dance.

   (ii) The piece is in $\frac{4}{4}$ time, as is most popular music.

   (iii) The piece is in $\frac{5}{4}$ time.

   (iv) The piece features an Afro-Cuban 3-2 son clave rhythmic pattern.

   (v) The piece is polyrhythmic.

   Answer: (v) The piece played was "Fake Empire" by *The National*, which features a 3-against-4 polyrhythm throughout.

   (b) An excerpt of a piece of music will be played. Which of the following best describes the musical style of the work heard?

   (i) Gregorian chant

   (ii) Early polyphonic music from the 1300's

   (iii) Bach chorale

   (iv) Neo-romantic (tonal but modern)

   (v) Atonal

   Answer: (iii) The piece played was the final chorale from Bach's *Wachet auf, ruft uns die Stimme*.

   (c) A musical scale will be played on the piano. Identify the type of scale played.

   (i) Major scale

   (ii) Minor scale

   (iii) Chromatic scale

   (iv) Whole tone scale

   Answer: (ii) A minor scale (harmonic) was played on the piano.

2. **Fill in the blanks:** You must show your work to receive any partial credit. (4 pts. each)

   (a) If two scales are considered identical when they contain the same set of notes (but not necessarily the same ordering of notes), then the number of **different** whole tone scales is <u>two</u>.

Answer: The number of essentially different whole tone scales is two because shifting the starting note of a whole tone scale to any other note produces the **same** set of notes by symmetry. Thus there are really only two different whole tone scales, one that starts on C and one that starts on C♯.

(b) The sum of the infinite series $\quad 9 - 3 + 1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots \quad$ is <u>27/4</u>.

Answer: This is an infinite geometric series that starts with $a = 9$ and has a ratio $r = -1/3$. The sum is therefore

$$S_\infty \;=\; \frac{a}{1-r} \;=\; \frac{9}{1-(-1/3)} \;=\; \frac{9}{4/3} \;=\; \frac{27}{4}.$$

(c) Suppose a measure of music contains a polyrhythm of 10 against 4. In order to see precisely where the pulse of each rhythmic pattern goes, the minimum number of pulses needed to subdivide the measure into is <u>20</u>.

Answer: The mathematical concept here is the least common multiple. The least common multiple of 10 and 4 is 20.

(d) If you start on the note D, go up a minor sixth and down a major third, you have arrived at the note <u>F♯ or G♭</u>.

Answer: If you begin on D and go up a minor sixth (8 half steps), you reach a B♭. Going down a major third (4 half steps) brings you to the note G♭ or F♯.

(e) The number of **black** keys on the modern piano keyboard is <u>36</u>.

Answer: There are 88 keys on the piano or roughly 7 octaves since $7 \cdot 12 = 84$. There are 5 black keys per octave so that gives $7 \cdot 5 = 35$. Finally, the remaining 4 keys (at the bottom of the piano) are a C, B, B♭ and A, which is one more black key for a total of 36.

3. **Rhythm:** You must show your work to receive any partial credit. (4 pts. each)

(a) How many sixteenth notes do you need to fill up a measure in $\frac{7}{4}$ time?

Answer: 28. Since the bottom number is a 4, the quarter note gets the beat. Since four sixteenth notes equal one quarter note, there are 4 sixteenth notes per beat. Finally, the top number indicates that there are 7 beats per measure, so we obtain $7 \cdot 4 = 28$ sixteenth notes in a measure.

(b) In $\frac{5}{8}$ time, a triple-dotted quarter note gets how many beats?

Answer: $3\frac{3}{4}$ beats. Since the bottom number is an 8, the eighth note gets the beat. Thus, a quarter note gets 2 beats. We then form a geometric series starting with 2 and with ratio $r = 1/2$ containing three more terms (one for each dot). This gives

$$2 + 1 + \frac{1}{2} + \frac{1}{4} \;=\; 3\frac{3}{4} \text{ or } \frac{15}{4}.$$

(c) In $\frac{3}{2}$ time, a measure begins with a dotted quarter rest. How many beats remain in the measure?

Answer: $2\frac{1}{4}$ beats. Since the bottom number is a 2, the half note gets the beat. Thus, a quarter rest takes up $1/2$ a beat and a dotted quarter rest takes up $1/2 + 1/4 = 3/4$ of a beat. Since the top number is a 3, we have $3 - 3/4 = 2\frac{1}{4}$ beats remaining in the measure.

4. **Least Common Multiple:**

   (a) Give an example of two integers a and b for which the $\text{lcm}(a, b) = ab$. (4 pts.)

   Answer: Any two integers which have no common factor (called relatively prime) will work. For example, $a = 2, b = 3$ or $a = 4, b = 7$ both satisfy $\text{lcm}(a, b) = ab$.

   (b) What condition on the $\gcd(a, b)$ must be satisfied in order for the least common multiple of a and b to be equal to ab? (5 pts.)

   **Answer:** The formula for the least common multiple is

   $$\text{lcm}(a, b) = \frac{ab}{\gcd(a, b)}.$$

   Therefore, we must have $\gcd(a, b) = 1$ in order for the least common multiple of a and b to be equal to ab.

5. **Scales:**

   (a) Write out a B major scale (ascending only) in both the treble and bass clef using correct accidentals – **no key signature**. Indicate the corresponding notes (use numbers) on the piano keyboard below. (8 pts.)
   Answer:
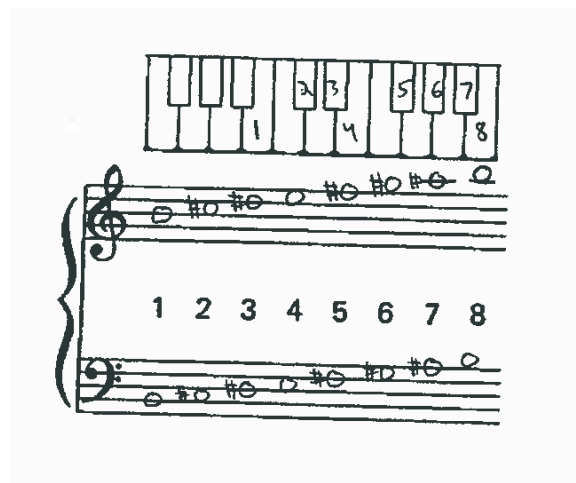
(b) Write out a G harmonic minor scale (ascending only) in both the treble and bass clef using correct accidentals – **no key signature**. Be sure to make the scale diatonic. (6 pts.)
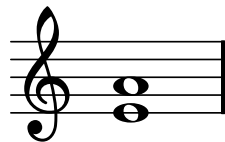
Answer:
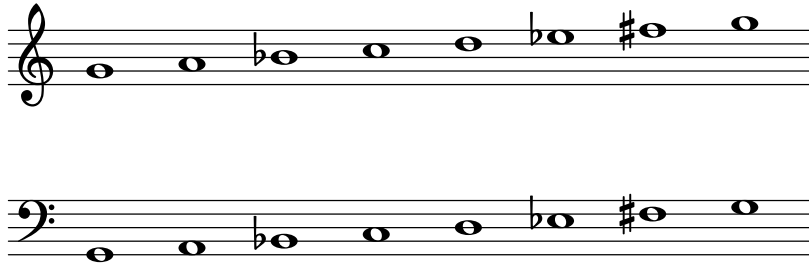
(c) What key has 6 flats? Write the flats for this key in the correct order. (6 pts.)
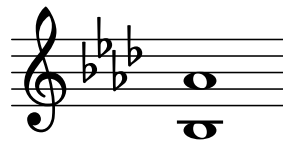
Answer: The key of G♭ has 6 flats. In order, they are B♭, E♭, A♭, D♭, G♭, C♭.
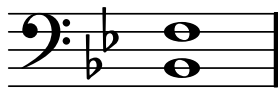
6. **Intervals:**

(a) Notate the given musical interval (including perfect, major and minor designation) below each of the following measures, (e.g., m2, M2, m3, M3, P4, etc.). Be sure to indicate your answers clearly. (2 pts. each)

Perfect 4th

minor 7th

Perfect 5th

minor 3rd

(b) What musical interval does the Holy Cross alma mater (sung to the tune "O Christmas Tree") open with? (3 pts.)

Answer: The Holy Cross alma mater opens with a perfect fourth.

7. **Transposition:** Answer the following questions based on the excerpt below.

(a) What key is the excerpt in and what number scale degree does it start on? (5 pts.)

Answer: The excerpt is in the key of A major. The starting note is an E which is the fifth scale degree in A major (A B C♯ D E ...).

(b) Using key signatures, transpose the entire excerpt into the key of B♭ major. (8 pts.)

Answer: