Mathematics and Music: Structure and Form

MONT 108N, MWF 9:00 - 9:50, Brooks Center 452, Fall 2011

Professor Gareth Roberts

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Office Hours: Mon. 1:00 - 2:00, Tues. 1:00 - 3:00, Wed. 8:00 - 8:50 (in Cool Beans) and 1:00 - 2:00, or by appointment.

Required Texts: *Music and Mathematics: From Pythagoras to Fractals*, edited by Fauvel, Flood and Wilson, and *Collapse* by Jared Diamond. You should also purchase the staff paper *Music Tablet*, available at the college bookstore.

Webpage: `http://mathcs.holycross.edu/~groberts/Courses/Mont1/homepage.html`
Homework assignments, handouts, schedule changes, exam materials, useful links and other important information will be posted at this site. Please bookmark it! The course page on Moodle will primarily be used for listening to audio files.

Common Area Designations: Mathematical Science or Arts

Course Content: The connections between mathematics and music from a structural perspective are plentiful. Both use a specialized form of notation to communicate their ideas. Each subject has its own logical structure and set of axioms finely tuned over centuries of study. Students in high school geometry learn the axiomatic technique of Euclid to write their first proofs. Students in music theory learning to compose for four voices are taught to avoid parallel fifths and octaves just as Bach did in his choral works. Mathematicians use numbers as the invariant building blocks of their theory as musicians use pitch as the common denominator of their creations. Just as the number three has the same abstract meaning to mathematicians everywhere, the concert pitch A440 used to tune modern orchestras is a global standard.

Composers and musicians, whether they are cognizant of the fact or not, use mathematical concepts in their creations. Bach frequently used mathematical operations such as translations and reflections in his fugues to create wonderfully rich counterpoint. Bell ringers in towers throughout England have been using permutations, symmetry and group theory in change ringing (sometimes called campanology) to announce important events since the early 1600's. Music theorists and mathematicians alike have claimed the appearance of the golden section in compositions of some of the great composers (Mozart, Debussy and Bartók, to name a few). The modern composer Xenakis used computers and probability theory to create his "stochastic music."

This course will explore the dynamic relationship between mathematics and music, investigating the inherent structures common to each discipline. Mathematical concepts and techniques will be utilized to gain a deeper understanding of music. The pertinent mathematical and musical concepts will be developed as the need arises. Musical topics to be considered include basic music theory (notation, rhythm, time signatures, pitch, scales, intervals, circle of fifths, chords, progressions), non-standard tunings and scales, and the overtone series. Mathematical topics to be covered include graphing functions, trigonometry, logarithms, equivalence relations, modular arithmetic, rational and irrational numbers, and continued fractions.

A tentative outline of the course is given below. Although we will cover material from several chapters of the primary course text, there will also be some handouts used for certain topics. In addition, a few days have been reserved for a course lab using a special type of monochord.

- Introduction to the course and Montserrat (1 class)
- Book Discussion: *Collapse* (4 classes)
- Rhythm: time signature, geometric series, subdividing, least common multiple (4 classes)
- Introductory Music Theory: notation, scales, circle of fifths, major and minor keys, intervals, tonality, transposition (7 classes)
- Exam I (in class)
- The Science of Music: a vibrating string, the sine function, the wave equation, beats, frequency versus length (monochord lab), the Pythagorean scale, the overtone series (8 classes)
- Guest Lecture: Sarah Webster (Studying the molecules of hearing) (1 class)
- Tuning and Temperament: just intonation, equal temperament, cents, rational and irrational numbers (5 classes)
- Guest Lecture: Shirish Korde (Rhythm and mathematics in classical Indian music) (1 class)
- Exam II (in class)
- Strähle's Guitar: Faggot's fretful fiasco, continued fractions (3 classes)
- Guest Lecture: Dan Hunt (Guitar construction) (1 class)
- Final Exam (Cumulative)

Course Objectives:

1. Investigate the multiple connections between mathematics and music.
2. Develop skills in critical thinking and abstract reasoning.
3. Develop an understanding of music theory and a deeper appreciation for music.
4. Integrate your artistic and analytical skills.
5. Have FUN while learning.
6. Participate in and contribute to the common events, lectures, etc. of the Natural World Cluster of the Montserrat program.

Important Dates: The following are **required** events:

- Trip to Harvard Forest: Sat., Sept. 17, 9:30 - 4:00
- Special Presentation on the HC Presidential Taskforce on the Environment: Wed., Sept. 21, 7:00 pm, Rehm Library
- Flute and Tabla Concert featuring Pat Spencer: Thurs., Nov. 17, 8:00 pm, Brooks Hall

Homework/Written Assignments: Homework will be assigned on a regular basis. Assignments will be posted on the course web page. While you are allowed and encouraged to work on homework exercises with your classmates, the solutions you turn in to be graded should be your own. Take care to write up solutions **in your own words**. There will also be a paper assigned on the book *Collapse*, and possibly some short written assignments related to the Natural World Cluster common events. On all assignments, plagiarism will not be tolerated and will be treated as a violation of the Departmental Policy on Academic Integrity.

Concert Reviews: You are required to attend two musical performances during the semester and turn in a typed, 1-2 page review of each concert. The purpose of these reviews is to enhance your musical appreciation, to support your fellow students and the arts, and to notice and describe any possible connections to course material. Your review should include basic information about the concert (location, date, performers, pieces, composers, etc.), your opinion of the concert (strengths and weaknesses) and some connection to course material. A schedule of upcoming concerts is linked from the course homepage.

Exams: There will be 2 midterm exams (in class) and a comprehensive final at the end of the semester. We will review for each midterm during the class period before the exam. If you have any specific learning disabilities or special needs and require accommodations, please let me know early in the semester so that your learning needs may be appropriately met. You will need to contact the director of Disability Services in Hogan 215 (x3693) to obtain documentation of your disability.

	Exam 1	**Fri., Oct. 21**	**9:00 - 9:50 am**
Exam Schedule:	**Exam 2**	**Wed., Nov. 30**	**9:00 - 9:50 am**
	Final	**TBA**	2.5 hours

Academic Integrity: The Department of Mathematics and Computer Science has drafted a policy on academic integrity to precisely state our expectations of both students and faculty with regards to cheating, plagiarism, academic honesty, etc. You are required to read this policy and sign a pledge agreeing to uphold it. A violation of the Departmental Policy on Academic Integrity will result in a 0 for that assignment (or exam) and a letter describing the occurrence of academic dishonesty will be sent to your Class Dean.

Grade: Your course grade will be based on the following breakdown:

- classroom participation/civility 5%
- concert reviews 5%
- homework (including any labs, worksheets and written assignments) 30%
- midterm exams 30%
- final exam 30%

How to do well in this course:

- Attend class, participate and ask questions. Be an engaged learner.
- Do your homework regularly.
- Work with your classmates.
- Ask for HELP when necessary.

> *May not Music be described as the Mathematic of Sense, Mathematics as the Music of reason? The soul of each the same! Thus the musician feels Mathematic, the mathematician thinks Music, — Music the dream, Mathematic the working life, — each to receive its consummation from the other.*
>
> James Joseph Sylvester, 1865

Natural World Cluster Description:

Humans have always sought to understand the world we inhabit and our place in it. Our sciences, arts and literature, philosophy and religion are all aspects of this search for understanding. Humans have the ability to describe and shape their environment, and therefore the environments of all other inhabitants of our planet. But what have been the consequences and what are our responsibilities? Our seminars this year will address questions such as: How do organisms and the environment interact in their intertwined paths of development? How are humans affecting the natural environment? How can mathematics be used to study both nature and aspects of human culture such as music? How has the idea of the frontier and the history of westward expansion shaped Americans' ideas about nature and our place in it? What do our choices of what and how we eat tell us about ourselves and our relation to the rest of the natural world? Finally, how do we deal with illnesses and other challenges to our well-being? Can we reconcile a belief in the goodness of the natural world with the presence of elements of that world that cause suffering?

Other Courses in the Cluster:

- Andrea Borghini – *I Eat, Therefore I Mean*, TR 12:30pm and TR 2:00pm
- Jodi Rymer – *Development and the Environment*, TR 9:30am
- John Little – *Modeling the Environment*, MWF 9:00am
- Stephanie Reents – *Go West, Young Man and Woman*, TR 9:30am
- Kelly Wolfe-Bellin – *Environmental Understanding*, WF 12:30pm