

MONT 110N Exam 1 SOLUTIONS

Math/Music: Structure and Form

October 20, 2010 Prof. G. Roberts

1. **Listening Questions:** Choose the best answer available. (5 pts. each)

(a) An excerpt of a piece of music will be played. Which of the following best describes the work heard?

- (i) The piece is in $\frac{2}{4}$ time invoking a lively dance.
- (ii) The piece is in $\frac{3}{4}$ time giving it a nice dance-like feel.
- (iii) The piece is in $\frac{5}{4}$ time as is most Latin music.
- (iv) The piece features an Afro-Cuban 3-2 son clave rhythmic pattern.
- (v) The piece is polyrhythmic.

Answer: (iv) The piece played was *Mueve La Cintura Mulata*, an example of Cuban son music that features the 3-2 son clave rhythmic pattern.

(b) An excerpt of a piece of music will be played. Which of the following best describes the musical style of the work heard?

- (i) Gregorian chant
- (ii) Early polyphonic music from the 1300's
- (iii) Bach chorale
- (iv) Neo-romantic (tonal but modern)
- (v) Atonal

Answer: (iv) The piece played was Samuel Barber's *Agnus Dei* which is a neo-romantic work with a tonal center of B minor, but with modern, often dissonant harmonies.

(c) A musical scale will be played on the piano. Identify the type of scale played.

- (i) Major scale
- (ii) Minor scale
- (iii) Chromatic scale
- (iv) Whole tone scale

Answer: (ii) A D harmonic minor scale was played.

2. **Fill in the blanks:** Work is only required to receive partial credit. (4 pts. each)

(a) The key of $\underline{G\flat}$ has 6 flats.

(b) How many sixteenth notes do you need to fill up a measure in $\frac{5}{8}$ time? 10

An eighth note gets the beat in $\frac{5}{8}$ time, so a sixteenth note gets half a beat. Thus, we need two sixteenth notes to complete one beat and $5 \cdot 2 = 10$ sixteenth notes to fill a measure with 5 beats.

(c) In $\frac{3}{2}$ time, a triple-dotted quarter note gets 15/16 beats.

In $\frac{3}{2}$ time a half note gets the beat, so a quarter note gets half a beat. Thus, a triple-dotted quarter note gets

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{15}{16} \text{ beats.}$$

(d) The sum of the infinite series $12 + 4 + \frac{4}{3} + \frac{4}{9} + \frac{4}{27} + \dots$ is 18.

This is an infinite geometric series with ratio $r = 1/3$ and first term $a = 12$. The sum is therefore

$$S = \frac{a}{1-r} = \frac{12}{1-1/3} = \frac{12}{2/3} = 18.$$

(e) In a measure of music containing a polyrhythm of 9 against 6, the minimum number of pulses needed to subdivide the measure in order to see precisely where each rhythmic pattern goes is 18.

To subdivide the entire measure into enough pulses to see where each of the 9 and 6 rhythmic patterns go, we need $18 = \text{lcm}(9, 6)$ pulses.

(f) If you start on the note C, go up a major sixth and down a minor third, you have arrived at what note? F \sharp

Using the C major scale, a major sixth up from C is the note A. To go down a minor third, count down three half steps to F \sharp .

(g) The number of **black** keys on the modern piano keyboard is 36.

This was a tricky one. There are 88 keys on the modern piano keyboard, which gives a little more than seven octaves. Since there are 5 black keys in an octave, we have $7 \cdot 5 = 35$ black keys. However, since $12 \cdot 7 = 84$, there are four keys unaccounted for. Only one of these is black, the B \flat at the bottom of the piano. This gives a total of 36 black keys.

3. Least Common Multiple:

- (a) Give an example of two integers a and b for which the $\text{lcm}(a, b) = ab$. (4 pts.)

Answer: Some examples include: $\text{lcm}(3, 4) = 12$, $\text{lcm}(3, 7) = 21$, $\text{lcm}(2, 5) = 10$.

- (b) What condition on the $\text{gcd}(a, b)$ must be satisfied in order for the least common multiple of a and b to be equal to ab ? (5 pts.)

Answer: Using the formula,

$$\text{lcm}(a, b) = \frac{ab}{\text{gcd}(a, b)},$$

we see that $\text{gcd}(a, b) = 1$ is a necessary condition for the $\text{lcm}(a, b) = ab$. In other words, a and b must have no common factors other than one.

- (c) If the condition in part (b) is satisfied, the numbers a and b are called relatively prime. (3 pts.)

4. Scales:

- (a) Write out an $A\flat$ major scale (ascending only) in both the treble and bass clef using correct accidentals – **no key signature**. Indicate the corresponding notes (use numbers) on the piano keyboard below. (8 pts.)

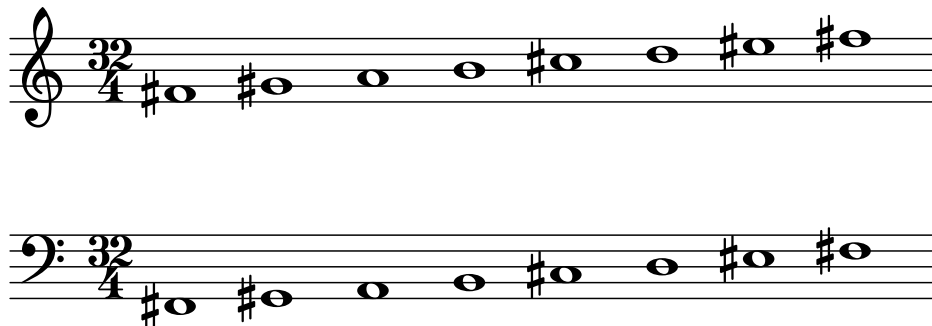
Answer: Using the major scale pattern WWHWWWH, we have

9. $A\flat$ major

The diagram shows an 8-key piano keyboard with fingerings 1 through 8 written above the keys. Below the keyboard is a handwritten musical score for the $A\flat$ major scale. The treble clef staff shows the notes $A\flat, B\flat, C, D, E, F, G, A\flat$ with fingerings 1, 2, 3, 4, 5, 6, 7, 8. The bass clef staff shows the notes $A\flat, B\flat, C, D, E, F, G, A\flat$ with fingerings 1, 2, 3, 4, 5, 6, 7, 8.

- (b) Write out an F \sharp harmonic minor scale (ascending only) in both the treble and bass clef using correct accidentals – **no key signature**. Be sure to make the scale diatonic. (6 pts.)

Answer: The pattern for a harmonic minor scale is WHWWH(1 1/2)H, we have



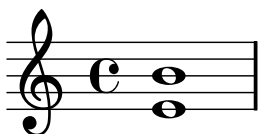
Note that in order to use a diatonic scale spelling (consecutive letter names), we need to use E \sharp rather than F, otherwise F is repeated twice and E is missed altogether.

- (c) What key has 5 sharps? Write the sharps for this key in the correct order. (6 pts.)

Answer: B major has 5 sharps. In order, they are F \sharp , C \sharp , G \sharp , D \sharp and A \sharp .

5. Intervals:

- (a) Notate the given musical interval (including perfect, major and minor designation) below each of the following measures. (eg. m2, M2, m3, M3, P4, etc.) Be sure to indicate your answers clearly. (8 pts.)



Answer: The interval on the left, from E up to B is a perfect fifth (P5). The interval on the right, from F \sharp up to B is a perfect fourth (P4).



Answer: The interval on the left, from B \flat up to A is a major seventh (M7). The interval on the right, from B \flat up to A \flat is a minor seventh (m7), one half step less than the previous interval.

- (b) What musical interval is the “heckle” interval, often heard at sporting contests sung by large groups of fans intending to mock opposing players with phrases like “air ball” or “Darryl”? (4 pts.)

Answer: The “heckle” interval is a minor third (m3).

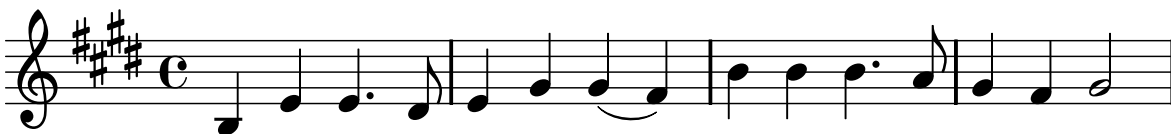
6. Answer the following questions based on the excerpt below.



- (a) What key is the excerpt in and what number scale degree does it start on? (5 pts.)

Answer: The piece is in the key of G major since the key signature has only one sharp. The excerpt starts on D which is the fifth scale degree in G major.

- (b) Using key signatures, transpose the entire excerpt into the key of E major. (8 pts.)



- (c) **Extra Credit:** Who composed the piece of music the excerpt is taken from?

Answer: The excerpt is from the famous Christmas carol *Hark! The Herald's Angels Sing*. In 1855, the English musician William Cummings adapted the music of Felix Mendelssohn to fit the original lyrics of Charles Wesley.