MATH 392: Seminar in Celestial Mechanics
Homework Assignment #1
DUE DATE: Thurs., Jan. 24, start of class.

Homework should be turned in at the BEGINNING OF CLASS. You should write up solutions neatly to all problems, making sure to show all your work. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your **own** work. Please cite any references (web based or text) that you may have used for assistance with the assignment.

Note: Please list the names of any students or faculty who you worked with on the top of the assignment.

1. Read Chapter 1 of *Celestial Mechanics: The Waltz of the Planets*, by Celletti and Perozzi.

   a) Suppose a satellite is orbiting the Earth. What terms are used to describe the position of the satellite when it is furthest from the Earth and closest to the Earth?

   b) Consider the graph and labels of the ellipse in Figure 1.4. Let c denote the distance from the center of the ellipse to a focal point (the sun, for example).

   i) The author's define the eccentricity of an ellipse to be the quantity $e = \dfrac{Q-q}{Q+q}$. Show that this is equivalent to $e = \dfrac{c}{a}$. Explain why $e = 0$ yields a circle. What happens to the shape of the ellipse as $e \to 1^-$?

   ii) Recall that an ellipse can be defined as the set of points such that the sum of the distances from the two foci is constant. Use this definition to find a relationship between the values a, b and c. Then find $\lim\limits_{c \to a^-} b$.

2. Prove the product rule for vector cross products. In other words, if $\mathbf{u}(t)$ and $\mathbf{v}(t)$ are two vector valued functions in $\mathbb{R}^3$, use standard rectangular coordinates to show that

$$\frac{d}{dt}(\mathbf{u} \times \mathbf{v}) = \dot{\mathbf{u}} \times \mathbf{v} + \mathbf{u} \times \dot{\mathbf{v}}.$$

3. Prove the vector identity

$$(\mathbf{u} \times \mathbf{v}) \times \mathbf{w} = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{v} \cdot \mathbf{w})\mathbf{u}$$

   stated in the middle of pg. 3 in Pollard's text.

4. Do the following exercises from Pollard's text: **2.1, 2.2, 3.1, 3.3**.

   Some hints: For **2.1**, choose a good location for the origin. You can assume that both attractive forces are proportional to the mass m. For **3.1**, try proof by contradiction and use equation (3.2) on pg. 6. For **3.3**, the term $(\mathbf{a} \times \mathbf{b})^2$ is really $(||\mathbf{a} \times \mathbf{b}||)^2$. There is a quick way and a long way to prove the first identity. Review your multivariable calculus for the quick way.