Final Project

Seminar in Celestial Mechanics, Spring 2008

The final assignment of the course is to complete a substantial project focusing on some particular aspect or application related to the course material. Your project will consist of both a typed report (roughly 10 pages) and an in-class presentation (25 minutes) during the final week of class. Your report can be written using Maple (which has nice word processing for mathematical symbols) or a regular word processing program with a hand-written appendix for mathematical formulae. You will be allowed to work in pairs for the project although it is expected that each member will contribute equally. The final project is worth 25% of your total course grade.

Timeline and Due Dates:

- March 27: Brief description of final project topic, including references and resources to be utilized
- April 10: Brief progress report detailing status of the project, including results and further lines of inquiry
- April 21: Title of Final Project along with names of group members
- April 24 – 29: Project Presentations (25 minutes)
- April 29: Final Report due (typed, roughly 10 pages)

Sample Topics:

The aim of the project is for you to explore in greater detail some aspect of the course you found interesting. Ideally, you will apply mathematical knowledge gained from this course, as well as others, to investigate a specific problem related to celestial mechanics. This may involve reading research papers and presenting the results and/or doing actual mathematics in order to investigate a topic, perhaps proving a few theorems along the way. This is not expected to be a ground-breaking research paper leading to publication, but rather a chance for you to delve deeper into a topic employing your well-developed mathematical abilities. Some sample topics are suggested below. Feel free to suggest your own topic if there is something different you would like to investigate.

Caution: Be careful when using material found on the Internet. For example, some of the information on *Wikipedia* is correct and some is not. Be sure to check your findings thoroughly by confirming them with at least two independent, published (ie. peer-reviewed) sources.

1. Consider the central force problem with a different gravitational law than the inverse square law. What happens if an inverse cube law is used instead? What are the similarities and differences? How do Kepler's laws change? Are there any physical applications for particular force laws? Are there certain laws for which the problem is completely solvable? What is the solution? This is a good opportunity to apply much of the mathematical material we used to "solve" the Kepler problem.

   Subjects involved: ODE's, Hamiltonian Systems, Physics

2. Investigate singularities in the n-body problem. What is a non-collision singularity and why is it important to celestial mechanists? Who came up with the first published example and what is it? What are the unresolved questions? What happens to the bodies as a total (or partial) collision is approached? Why? What does it mean to say a collision can be *regularised*? There is plenty of complicated mathematics here so a good goal would be to present known results, perhaps proving a few along the way, and give a summary of relevant information.

   Subjects involved: ODE's, Analysis, Physics

3. Examine the Titius-Bode law predicting astronomical distances in our solar system (discussed in Chapter 5 of Celletti and Perozzi). Perform a detailed statistical analysis describing the usefulness of this law. Are there other types of curves which fit the data better? If so, use the method of least squares to find the best fit. What are their errors? Is it possible to find similar algebraic rules to describe other distances in the solar system, such as the satellites of Jupiter and Saturn? Based on your findings, draw some conclusions on the usefulness of these laws.

   Subjects involved: Astronomy, Statistics, Modeling

4. Explore the integral manifolds of the spatial and planar three-body problems. These "level surfaces" are obtained by fixing the values of the 10 integrals of motion (6 in the planar case) and then examining the possible configurations that can be obtained. This set of positions is where the motion can take place. What parameters do the integral manifolds depend on? What role do central configurations play? What mathematical concepts are used to describe the different types of integral manifolds? (Part of your project will involve trying to understand what these concepts mean.) What is the geometry of these manifolds? Where do bifurcations occur?

   Subjects involved: ODE's, Hamiltonian Systems, Geometry, Algebraic Topology

5. Investigate relative equilibria in the n-body problem. What kinds of configurations are possible? How does symmetry play a role? Which relative equilibria have been used to model phenomena in our solar system? Which are linearly stable and how do the values of the masses effect stability? What type of techniques do researchers use to study relative equilibria. Describe some examples.

   Subjects involved: ODE's, Hamiltonian Systems, Analysis, Geometry, Topology

6. Possible Computer Programs: These could be written using Maple or some programming language you may be comfortable with (such as `C++`). Your project should include your code as well as some explanations on how it was created. For your presentation, live demonstrations of your program would be great.

   - Write a program for the Kepler problem that, given random initial conditions, calculates the pertinent orbital parameters, graphs the solution over time and calculates the value of u as the orbit evolves.
   - Write a program that simulates solutions in the n-body problem. Start with $n = 3$. What techniques are used to numerically compute the solution? Are the integrals of motion conserved during a solution? This project requires some knowledge of numerical analysis.
   - Write a program that searches for relative equilibria in the n-body problem. Start with $n = 3$. Given a set of masses, numerically locate the relative equilibria. Can you ever be sure you found them all? Provide a plot of each solution found.