Seminar in Mathematics and Climate
Homework Assignment #2
Due Thurs., Feb. 15, start of class

You should write up solutions neatly to all problems, making sure to show all your work. A nonempty subset will be graded. You are encouraged to work on these problems with other classmates, and it is ok to use internet sources for help if it's absolutely necessary (with proper citation); however, the solutions you turn in should be your own work and written in your own words.

Note: Please list the names of any students or faculty who you worked with on the assignment.

1. Read the two articles "Snowball Earth," Paul F. Hoffman and Daniel P. Schrag, *Scientific American*, Jan. 2000, pp. 68–75 and "'Snowball Earth' Might Have Been Slushy," Michael Schirber, *Astrobiology Magazine*, Aug. 2015 (4 pp).

   (a) Write a paragraph or two explaining the Snowball Earth theory as outlined by Hoffman and Schrag. What evidence do they provide to support their theory? Why is it important that the continents were clustered near the equator?

   (b) According to Schirber, what are the main arguments against a complete runaway snowball event? What role does the ocean play? How does this effect the search for life on planets outside our solar system?

   (c) In terms of the Budyko Energy Balance Model we have been discussing in class, which parameter(s) would you change (and by how much) to model the conditions of the Neoproterozoic era? How might you adjust the albdeo to account for a slowly moving ice sheet?

2. Recall **Archimedes' Hat-Box Theorem:** Let $L_1 = R\sin\theta_1$ and $L_2 = R\sin\theta_2$ be two "latitudes" on a sphere of radius R, with $L_1 > L_2$. Then the surface area enclosed by L_1 and L_2 is $2\pi R(L_1 - L_2)$.

   (a) Prove the theorem by using the formula for the surface area of a surface of revolution

   $$S = 2\pi \int_a^b f(x)\sqrt{1 + f'(x)^2}\, dx. \qquad (1)$$

   Hint: What function do you revolve around the x-axis to obtain a sphere of radius R?

   (b) Suppose we choose coordinates so that the Earth corresponds to the unit sphere ($R = 1$). Find the latitude θ (in degrees) for which half the Earth's surface area lies between $-\theta^\circ$ and θ°.

   (c) The sphere and the cylinder are the only two surfaces of revolution for which surface area is equivalent to $2\pi R$ times the change in height (see Figure from class lecture on 2/13). In the language of differential geometry, we say that the projection from the cylinder to a sphere of the same radius is an *isometry* (preserves area). We can prove this fact using formula (1).

Assume that $R = 1$ and let $y(t)$ be the function to be revolved around the t-axis. The surface area obtained by rotating $y(t)$ around the t-axis over the interval $a \le t \le x$ is given by

$$S(x) \;=\; 2\pi \int_a^x y(t)\sqrt{1 + y'(t)^2}\, dt,$$

where x is the primary variable. We want to find a function $y(t)$ such that $S(x) = 2\pi(x - a)$. Use the Fundamental Theorem of Calculus to derive the ODE

$$\frac{dy}{dt} \;=\; -\sqrt{\frac{1}{y^2} - 1}\,, \quad y(0) = 1$$

for $y(t)$. Solve the ODE and show that there are *two* solutions: one corresponding to a cylinder and the other corresponding to the unit sphere. (Thanks to Prof. Hwang for suggesting this problem.)