Seminar in Mathematics and Climate
Homework Assignment #1
Due Thurs., Feb. 8, start of class

You should write up solutions neatly to all problems, making sure to show all your work. A nonempty subset will be graded. You are encouraged to work on these problems with other classmates, and it is ok to use internet sources for help if it's absolutely necessary (with proper citation); however, the solutions you turn in should be your own work and written in your own words.

Note: Please list the names of any students or faculty who you worked with on the assignment.

1. Read Chapter 1 from the course textbook *Mathematics and Climate*, by Kaper and Engler.

   (a) Explain the difference between *climate* and *weather*.

   (b) What is the IPCC and why is it important? Look it up on the Internet and find out how many people contributed to the 5th assessment report, published in 2014.

   (c) The authors describe their approach to climate modeling as a *system-level approach*. What do they mean by this?

   (d) Do Exercise 1 on p. 10. Give an example of a positive feedback loop and another example of a negative feedback loop.

2. Consider the differential equation $\dfrac{dy}{dt} = 3y^3 - 12y^2$.

   (a) Sketch the phase line and identify each equilibrium point as a sink, source, or node.

   (b) On the same set of coordinate axes, plot four solutions $y(t)$ obtained from the following initial conditions: $y(0) = -1$, $y(0) = 0$, $y(0) = 3$, and $y(1) = 3$.

3. Consider the differential equation $\dfrac{dy}{dt} = Ay - y^3$, where $A \in \mathbb{R}$ is a parameter.

   (a) Find the equilibrium points.

   (b) For what value(s) of A does a bifurcation occur? Describe in words the qualitative behavior of solutions (e.g., number and type of equilibria) before, at, and after the bifurcation.

   (c) Sketch the bifurcation diagram for varying A, including several different phase lines. Plot A on the horizontal axis and y on the vertical axis. Stable equilibria should be drawn as a solid curve, while unstable equilibria should be dashed.

4. Read Chapter 2 from the course textbook *Mathematics and Climate*, by Kaper and Engler. Most of this material was covered on the first computer project and in the lectures, although they do a much better job explaining the Greenhouse Effect than I did (see Section 2.4).

   (a) The authors present a modification to the energy balance model we have been studying by replacing the Stefan-Boltzmann law with a new function. What type of function do they use and who first suggested this idea? What are the advantages of this approach?

(b) Do Exercise 8 on p. 24. You can ignore the "suitable linearization" instructions in part (i).

(c) Do Exercise 11 on pp. 25–26. *Hint:* Draw a phase line for the ODE before you start to sketch solutions.