Seminar in Mathematics and Climate

MATH 392-01, Spring 2018

TuTh 11:00 - 12:15, Swords 330

Professor Gareth E. Roberts

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Office hours: Mon. 10:30 - 12:00, Tues. 1:00 - 1:50, Wed. 1:00 - 2:00, Thurs. 9:45 - 10:45, or by appointment.

Required Text: *Mathematics and Climate*, Hans Kaper and Hans Engler, Society for Industrial and Applied Mathematics (2013) (a copy is on reserve in the Science Library)

Course Prerequisite: MATH 241 (Multivariable Calculus) and MATH 244 (Linear Algebra)

Web page: `http://mathcs.holycross.edu/~groberts/Courses/MA392/homepage.html`
Homework assignments, computer projects, useful links and other important information will be posted at this site. Please bookmark it!

Course objectives:

- Learn, apply, and synthesize the mathematical techniques used in climate science.
- Develop skills to understand and modify mathematical models.
- Conduct a detailed investigation into a research topic concerning the mathematics of climate.
- Work and communicate with your peers.
- Have FUN applying your mathematical skills to an important discipline.

Syllabus: The goal of this course is to learn and apply some of the mathematical techniques pertinent to the field of climate science. Climate is measured by averaging the weather over space and time, typically over long time periods such as decades, centuries, or millennia. Understanding the Earth's climate system and how it responds to physical changes (e.g., shifts in the Earth's orbit) and human influence (e.g., increasing greenhouse gases) is one of the most important problems of our time. Controlled physical experiments of the climate are impossible; the only available approach is to use mathematical models, computer simulations, and data analysis.

We will consider the following types of questions: Is the planet warming and are we responsible? What determines the Earth's surface temperature and how does it vary by latitude? What are some of the low-dimensional mathematical models used to understand climate and how do they depend on various parameters? What are the typical steady states for the Earth's climate system and how have these evolved over the history of the planet? Why does the carbon dioxide in the atmosphere affect the climate? What are "tipping points" and how do mathematicians study them? How do changes in the Earth's celestial orbit affect the climate? Why has the Earth cycled through ice ages over the last million years? What aspects of climate can be reasonably predicted and over what time periods?

Our study is fundamentally an interdisciplinary one, bringing in concepts from physics, chemistry, geology, environmental studies, computer science, statistics, and applied mathematics. The mathematical theory and techniques we will study will be motivated by the climate science. Expected areas of emphasis include differential equations, dynamical systems, calculus, linear algebra, and statistics. It is not expected that you are already proficient in these fields. The requisite mathematical tools and concepts will be covered as needed.

This seminar is in the Applied Math breadth area and is designated as a "project course." It has also been approved for the Environmental Studies major. A key feature of the course is the completion of a substantial final project exploring some topic related to mathematics and climate. In theory, the mathematical skills you develop in this course, as well as those acquired from other courses taken in the department, will allow you to make an in-depth investigation into your research topic.

Specific topics to be explored and class events include:

- Evidence and consequences of global warming, conceptual climate modeling, the greenhouse effect, silicate weathering
- Energy balance models, insolation, albedo, Stefan-Boltzmann law
- Budyko's model, the ice-albedo feedback, ice-line dynamics, the Budyko-Widiasih model, snowball Earth
- Bifurcation theory, tipping points, examples (saddle-node, hysteresis, cusp catastrophe, Hopf, pitchfork)
- Oceans and climate, box models, Stommel's model, equilibria and stability of a 2D system
- Climate and statistics, regression analysis, Keeling's CO_2 curve
- Glacial cycles, Fourier transforms, Milankovitch theory, ice-core data, the 100,000- and 41,000-year mysteries
- Field trip to **Harvard Forest** (sometime in mid-April)
- Biosphere and climate, the carbon cycle, carbon in the ocean, algal blooms
- Good and bad science in the film "The Day After Tomorrow," Dansgaard-Oeschger events, heat imbalance

Homework: There will be several homework assignments (problem sets) given out during the course of the semester. Assignments will contain exercises from the Kaper and Engler textbook as well as some questions based on distributed readings. These will be posted on the course webpage. While you are allowed and encouraged to work on homework problems with your classmates, the solutions you turn in to be graded should be your own. If you use the Internet for help on homework, be sure to cite the website(s) visited. Take care to write up solutions **in your own words**. Plagiarism will not be tolerated and will be treated as a violation of both the departmental policy on academic integrity and the college's policy on academic honesty.

NOTE: Late homework will not be accepted. However, you will be allowed ONE "mulligan" over the course of the semester where you can turn in the assignment up to one week after the original due date.

In-class Presentations: During the semester you will be expected to give a few in-class presentations demonstrating the solution to a particular homework problem (likely on the day the homework is due.) These presentations will be brief and are designed to enhance your oral communication skills as well as prepare you for your final project presentation.

Computer Projects: There will be 3–4 computer projects assigned over the course of the semester requiring the use of Matlab or some other type of mathematical software. These projects are numerical "experiments" designed to discover or reinforce important concepts discussed in class. The first project will introduce the process of mathematical modeling by building and refining a simple energy balance model for the temperature of the Earth. Another project will involve data analysis and fitting curves to real data such as CO_2 averages and oxygen isotopes from coral reefs. Projects will be carried out in groups of 2 to 3 people with one typed set of solutions to be turned in for the whole group.

Final Project: You are required to complete a substantial final project (working in small groups) focusing on some particular aspect or application related to the course material. Details and suggestions of topics will be distributed early in the semester. You may find a topic of interest as you peruse the text by Kaper and Engler. Your project should include the creation or adjustment of a mathematical model and/or analysis of real world data. It is expected that you will adhere to the posted deadlines as your project develops (e.g., proposal, progress report, first draft). Your project will include both a written report and an in-class presentation during the final week of class.

Academic Integrity: The Department of Mathematics and Computer Science has drafted a policy on academic integrity to precisely state our expectations of both students and faculty with regards to cheating, plagiarism, academic honesty, etc. You are required to read this policy and sign a pledge agreeing to uphold it. A violation of the Departmental Policy on Academic Integrity will result in a 0 for that assignment and a letter describing the occurrence of academic dishonesty will be sent to your Class Dean.

Diversity and Inclusion: It is my intent that students from all diverse backgrounds and perspectives be well-served by this course, that students' learning needs be addressed both in and out of class, and that the diversity that students bring to this class be viewed as a resource, strength, and benefit. Any suggestions you have pertaining to diversity and inclusion are encouraged and appreciated.

Grade: Your course grade will be determined by the scores you receive for each of the following items:

- classroom participation/in-class presentations 10%
- homework 30%
- computer projects 20%
- final project 40%

How to do well in this course:

- Attend class, participate and ask questions. Be an active learner.
- Do your homework regularly.
- Read the text. (Yes, this is possible!)
- Work with your classmates.

"We wouldn't be able to start understanding climate change if it wasn't for our understanding of applied math." — Emily Shuckburgh (climate scientist, British Antarctic Survey)