The Greenhouse Effect

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Lecture Outline

- Observations from Computer Project #1: Bifurcations and Hysteresis
- The Greenhouse Effect
- Ice Core Data and Human-Induced Climate Change

Computer Project #1: Observations

- Modeling is an iterative process. Sometimes the results are poor or counter-intuitive. Adjust your model.
- Units are important (e.g., working in kelvin or Celsius, metric system or not).
- There are typically lots of parameters ($\sigma, Q, \alpha, \epsilon$) and the outcome of a model can vary greatly even for a small change in the value of a parameter—bifurcations.
- Tuning: Sometimes we adjust the parameters to make our model agree with known data. This looks good, but can also be misleading to those evaluating the model.
  Example: Climate Model #3 introduced ϵ to model the greenhouse effect and obtain the current average temperature of the Earth. No physics used at all: $Q(1-\alpha) = \epsilon\sigma T^4$

Climate Model #5

$$
\begin{aligned}
C\frac{dT}{dt} &= E_{\text{in}} - E_{\text{out}} \\
&= (1-\alpha(T))Q - \epsilon\sigma T^4
\end{aligned}
$$

where

$$
\begin{aligned}
T &= \text{global average surface temperature, in K} \\
\alpha(T) &= 0.7 - 0.4\frac{e^{(T-265)/5}}{1+e^{(T-265)/5}} \quad \text{(albedo)} \\
Q &= 1/4 \text{ of the solar constant } S\text{, } 342\ \mathrm{W/m^2} \\
\sigma &= 5.67\cdot 10^{-8}\ \mathrm{W/(m^2\cdot K^4)} \\
\epsilon &= \text{greenhouse effect parameter}
\end{aligned}
$$

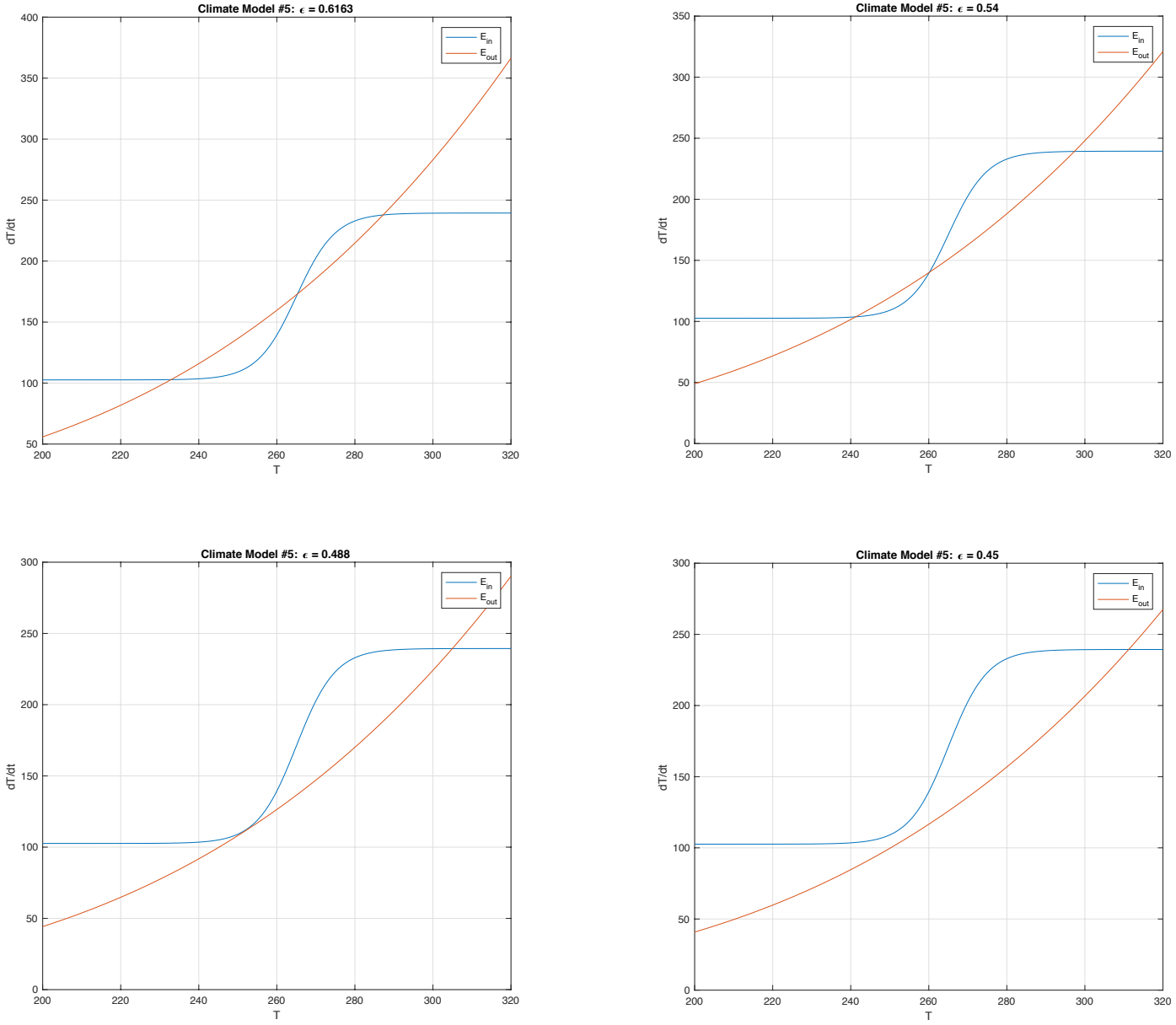


Figure: The bifurcation that arises when decreasing ϵ in Climate Model #5.

Climate Model #5: Bifurcation #1

Bifurcation value: $\epsilon_h \approx 0.488$

- For $\epsilon > \epsilon_h$, there are three equilibrium temperatures. The largest (warm, current climate) and the smallest (frigid, snowball state) are stable (sinks).
- At $\epsilon = \epsilon_h$ (a saddle-node bifurcation), the two smaller equilibria merge into one, forming a node. The larger equilibrium point has increased in value (to approximately 305 K).
- For $\epsilon < \epsilon_h$, there is only one equilibrium temperature corresponding to a very warm planet (a hothouse over 305 K $\approx 32^\circ$ C).

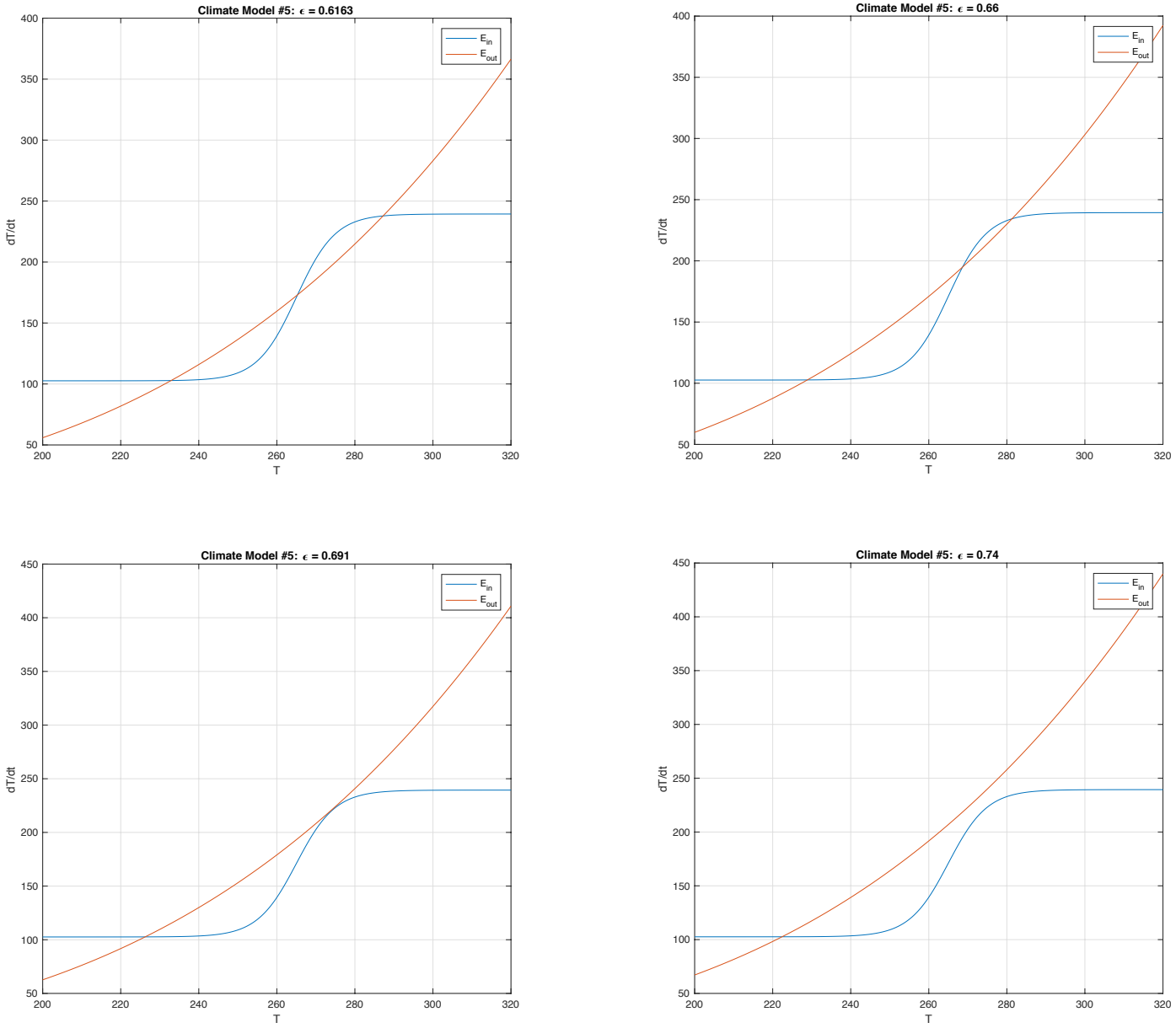


Figure: The bifurcation that arises when increasing ϵ in Climate Model #5.

Climate Model #5: Bifurcation #2

Bifurcation value: $\epsilon_{sb} \approx 0.691$

- For $\epsilon < \epsilon_{sb}$, there are three equilibrium temperatures. The largest (warm, current climate) and the smallest (frigid, snowball state) are stable (sinks).
- At $\epsilon = \epsilon_{sb}$ (a saddle-node bifurcation), the two larger equilibria merge into one, forming a node. The smaller equilibrium point has decreased in value (to approximately 225 K).
- For $\epsilon > \epsilon_{sb}$, there is only one equilibrium temperature corresponding to a very frigid planet (Snowball Earth, less than 225 K $\approx -48^\circ$ C). Amazingly, there is evidence that Earth was in this state about 630 Mya (million years ago) and 715 Mya.
- The bifurcations at ϵ_h and ϵ_{sb} are examples of tipping points, where it is suddenly possible to move from one climate state to another. The bifurcation diagram demonstrates the phenomenon of hysteresis.

The Greenhouse Effect

Recall: Climate Model #2 (incorporating albedo α but no greenhouse effect $\epsilon = 0$) predicts the Earth's temperature to be 254.9 K $= -18.25^\circ$ C (snowball!) This would be the temperature of the Earth without the greenhouse effect.

The Greenhouse Effect: Greenhouse gases (carbon dioxide = CO2, water vapor = H2O, methane = CH4) are transparent to visible light, but opaque to infrared light.
The energy from the sun passes through the atmosphere and heats the surface. The surface radiates energy at a lower temperature (infrared), which is absorbed by the atmosphere.

Who discovered the greenhouse effect? A mathematician!
Joseph Fourier (1827), Mémoire sur les Températures du Globe Terrestre et des Espaces Planétaires, *Mémoires de l'Académie Royale des Sciences*, t. vii., p. 569.

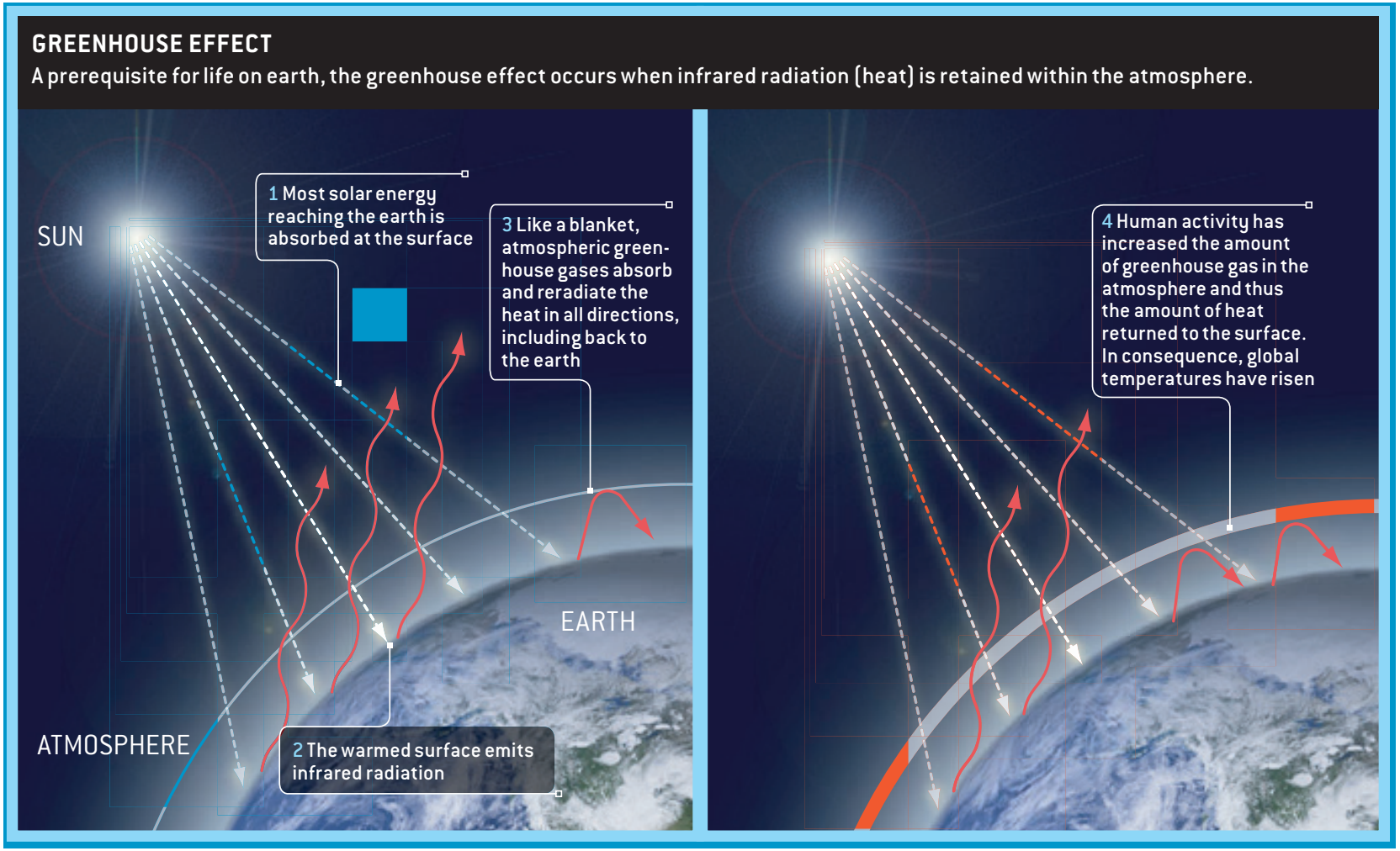
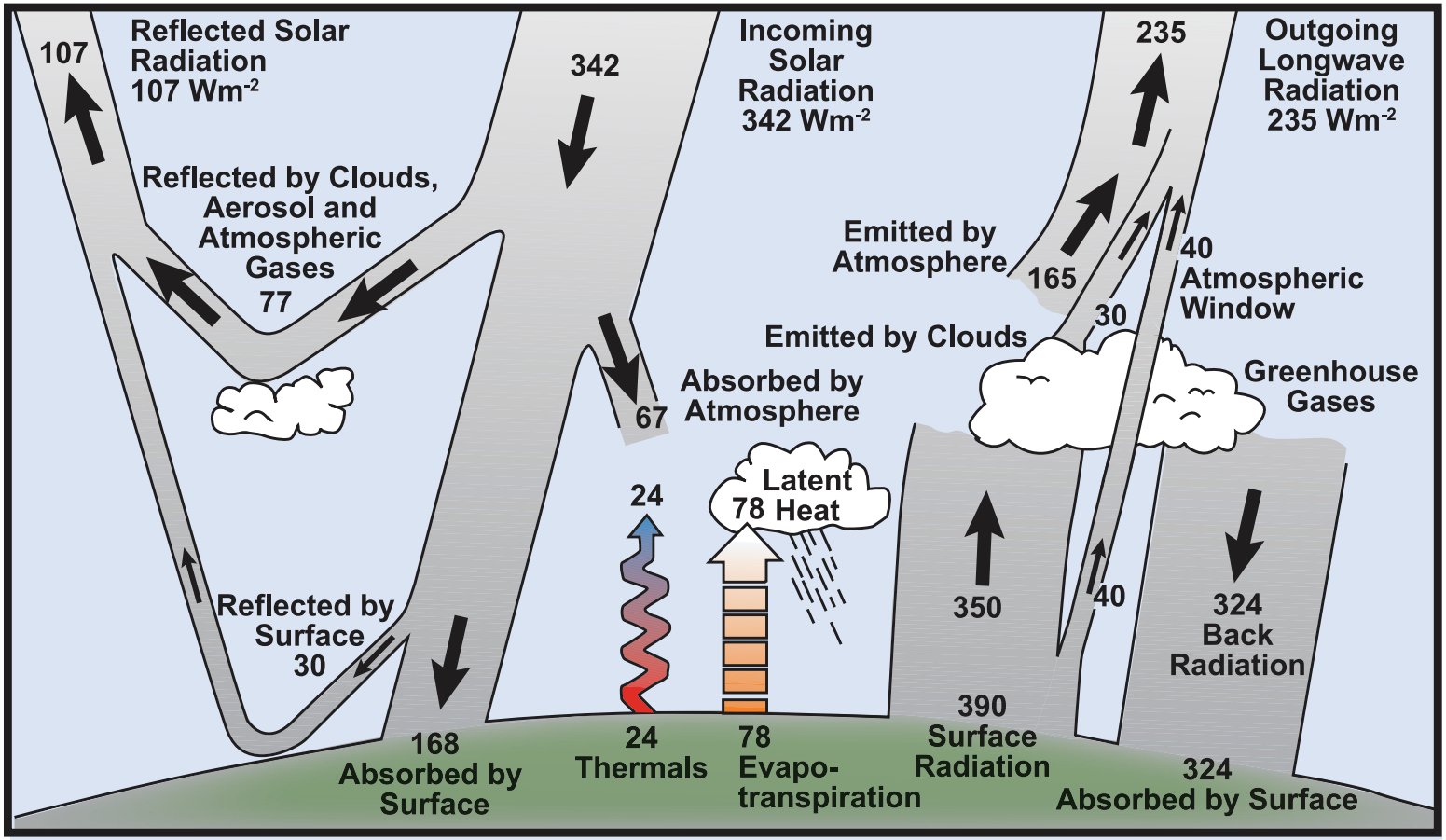


Figure: Overview of the greenhouse effect. Source: "A Climate Repair Manual," Gary Stix, *Scientific American*, Sept. 2006, 46–49.

The Greenhouse Effect: Origins

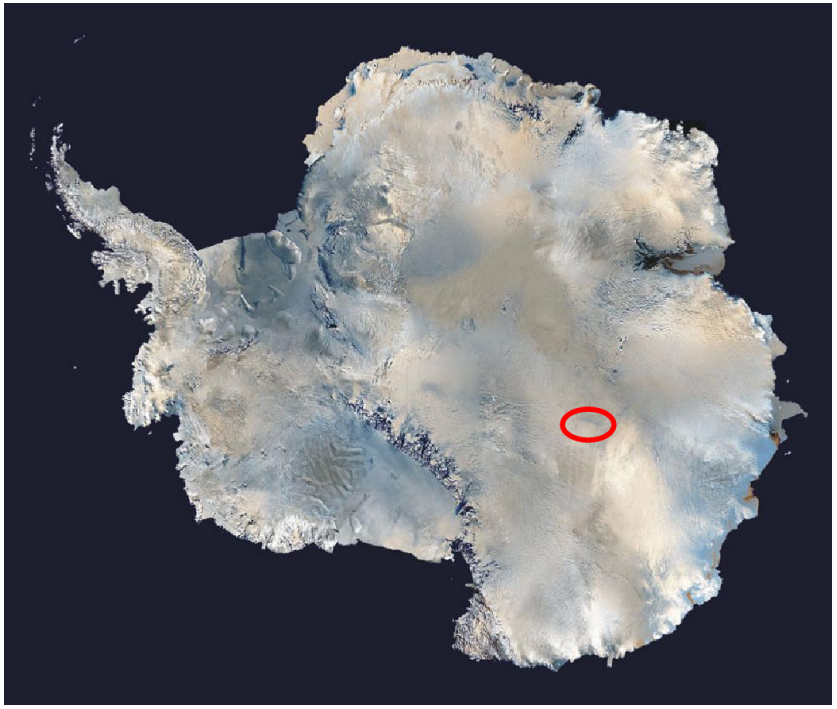
- Theory well formulated by the nobel-prize winning Swedish chemist Svante Arrhenius (1859–1927).
- "On the Influence of Carbonic Acid in the Air upon the Temperature of the Ground," *Philosophical Magazine and Journal of Science* (Fifth Series) 41, pp. 237–276, 1896.
- Made extensive calculations on the effect that increasing the amount of CO2 in the atmosphere would have on the Earth's surface temperature—calculations that have lasted until this day. His predictions agree with those obtained from current super-computer global climate models.



FAQ 1.1, Figure 1. *Estimate of the Earth's annual and global mean energy balance. Over the long term, the amount of incoming solar radiation absorbed by the Earth and atmosphere is balanced by the Earth and atmosphere releasing the same amount of outgoing longwave radiation. About half of the incoming solar radiation is absorbed by the Earth's surface. This energy is transferred to the atmosphere by warming the air in contact with the surface (thermals), by evapotranspiration and by longwave radiation that is absorbed by clouds and greenhouse gases. The atmosphere in turn radiates longwave energy back to Earth as well as out to space. Source: Kiehl and Trenberth (1997).*

Figure: Heat Balance. Recall: $Q = S/4 = 342\ \mathrm{W/m^2}$. Source: "Historical Overview of Climate Change Science," IPCC AR4, (2007) p. 96.

Understanding the Earth's Past Climate



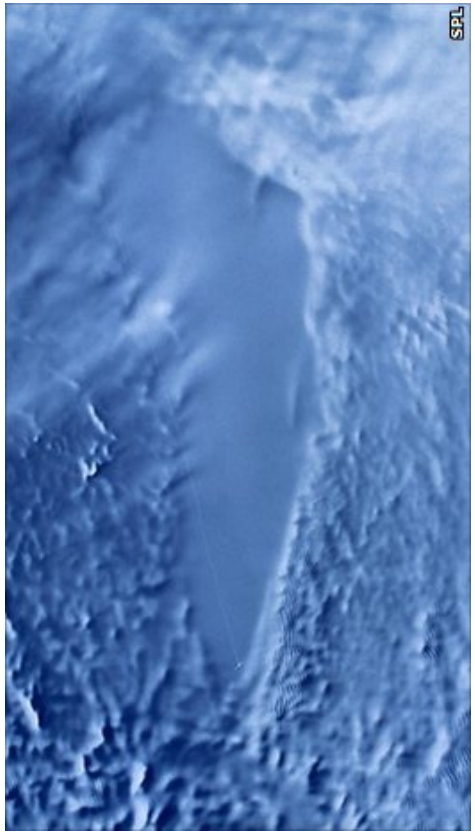


Figure: Lake Vostok, Antarctica.

Ice Cores

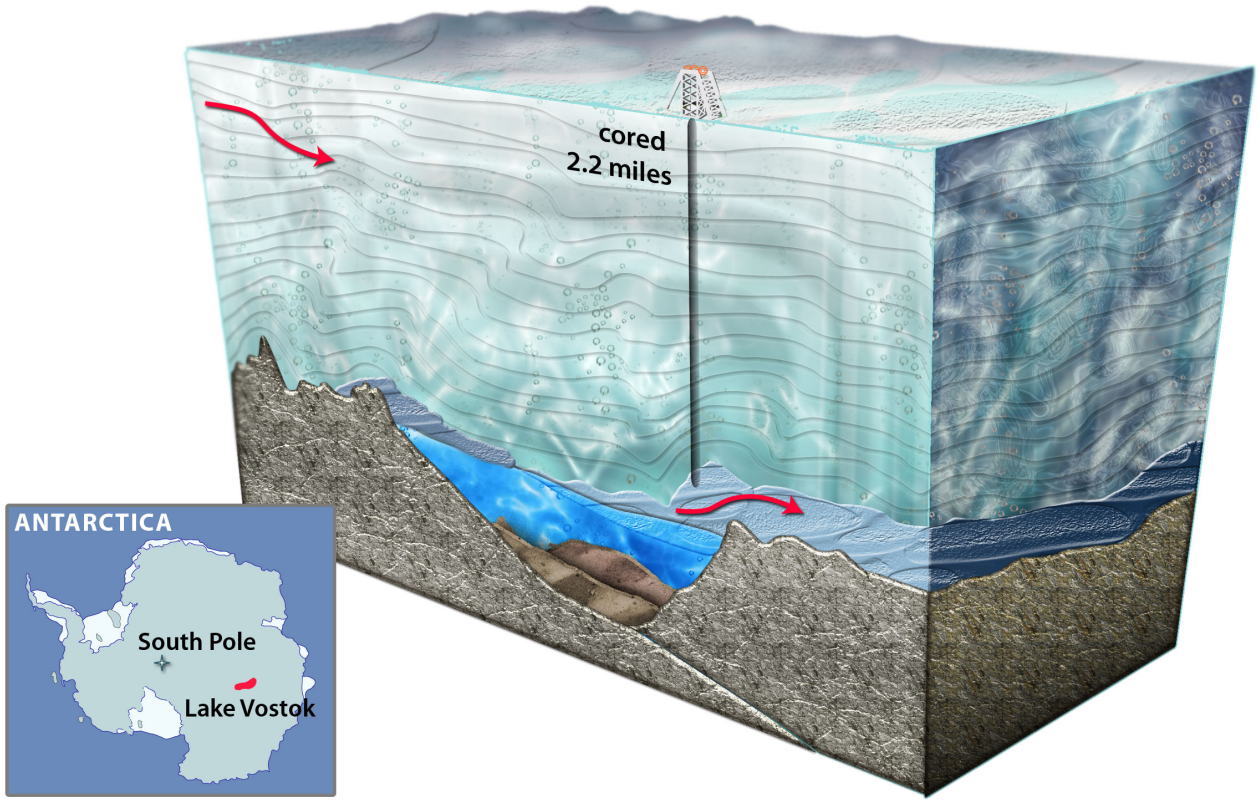


Figure: Some ice cores contain ice 800,000 years old (carbon dating). The proportions of different oxygen and hydrogen isotopes help reconstruct ancient temperatures; air trapped in bubbles can be analyzed to determine past levels of greenhouse gases.

Figure: The GISP2 (Greenland Ice Sheet Project #2) drill tower in central Greenland. Great read: “The Two-Mile Time Machine: Ice Cores, Abrupt Climate Change, and Our Future,” Richard Alley, Princeton University Press (2000).

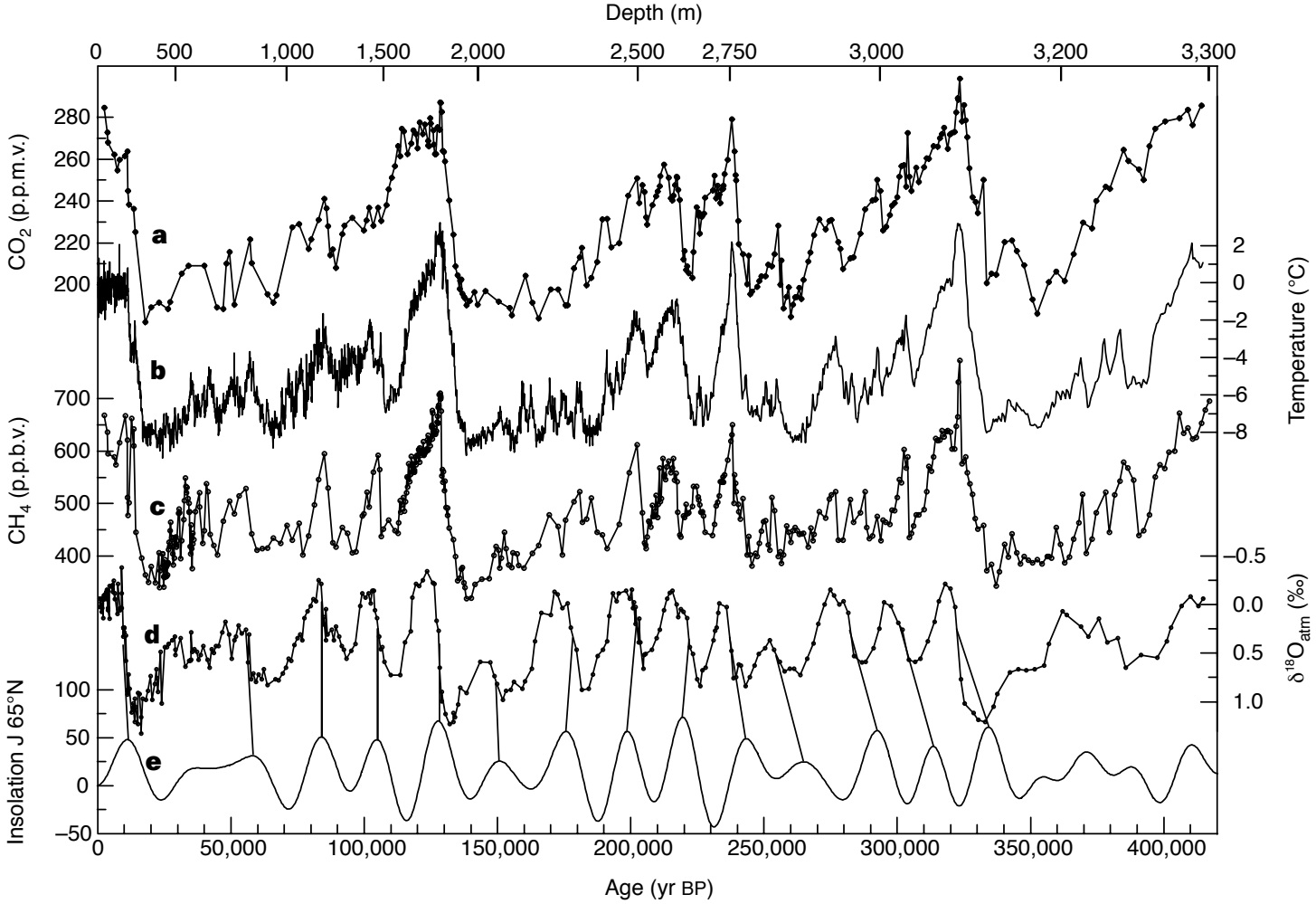


Figure: Time series data from ice core samples in Lake Vostok, Antarctica. **a** = CO2, **b** = Antarctic temp., **c** = Methane. Source: "Climate and atmospheric history of the past 420,000 years from the Vostok ice core, Antarctica," Petit, et. al., *Nature* **399**, June 3, 1999, pp. 429–436.

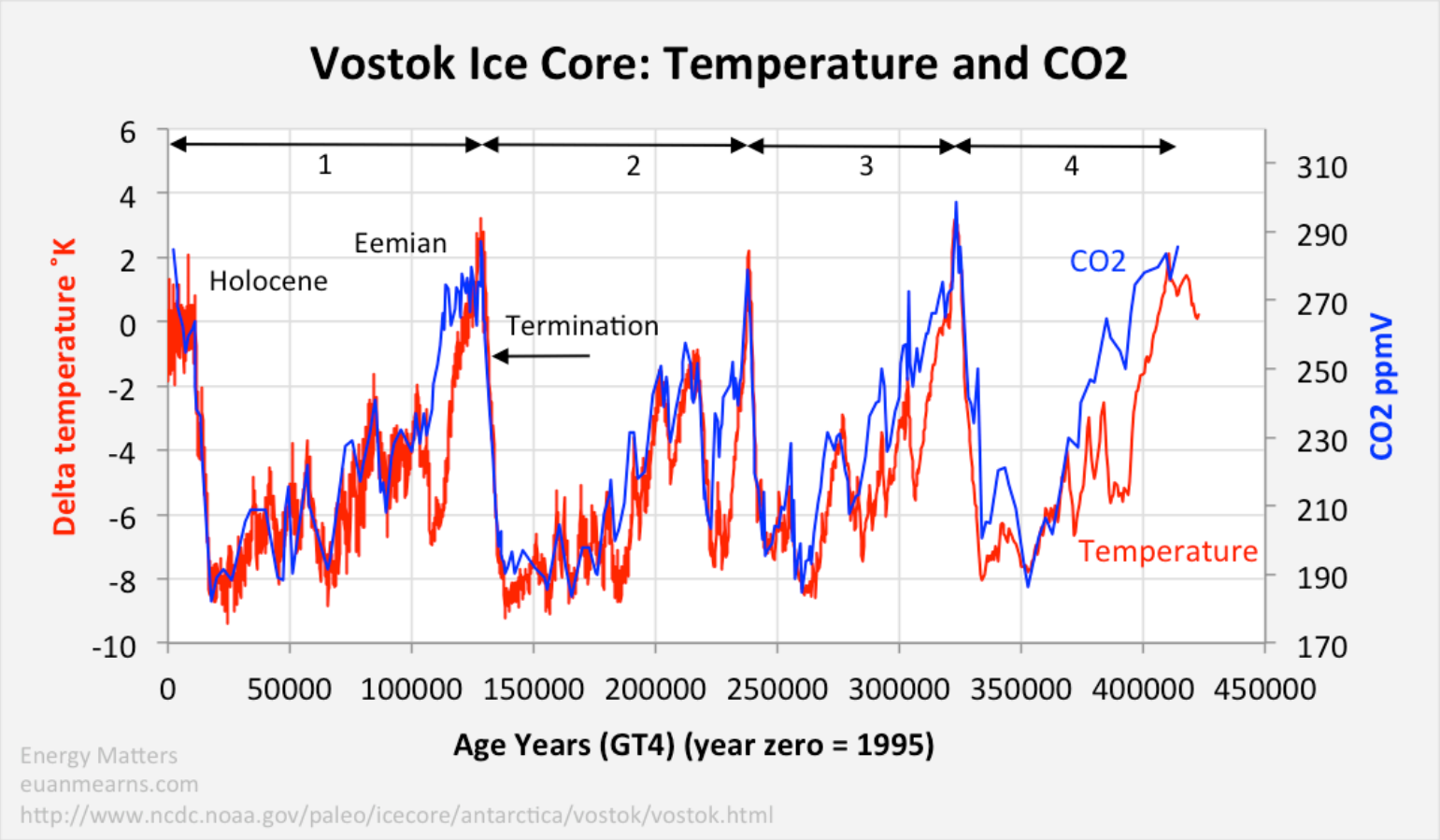


Figure: Very strong correlation between temperature and CO2 concentrations from the Vostok ice core data. Concentrations over the last 420,000 years range from 185 to 300 ppmV (parts per million by volume).

Cause for Concern: Human-Induced Warming

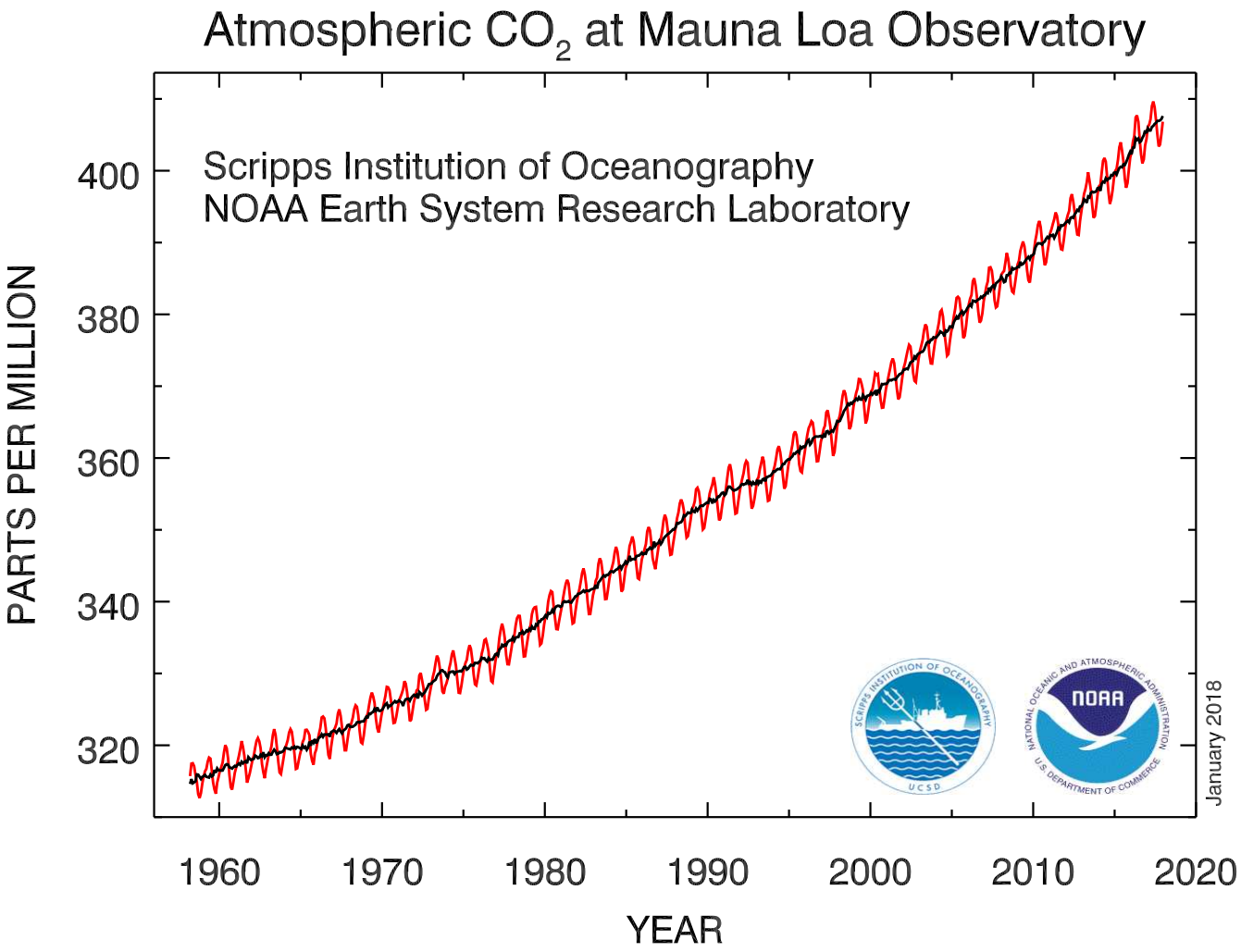


Figure: CO2 concentrations measured from the Mauna Loa Observatory in Hawaii as of January 2018. Source: NOAA Earth System Research Laboratory, Global Monitoring Division

Cause for Concern: Human-Induced Warming

- Human activity since the Industrial Revolution (e.g., burning of fossil fuels) has led to massive increases in greenhouses gases.
- "atmospheric concentrations of carbon dioxide, methane and nitrous oxide are unprecedented in at least the last 800,000 years. Their effects, together with those of other anthropogenic drivers, have been detected throughout the climate system and are extremely likely to have been the dominant cause of the observed warming since the mid-20th century." — IPCC Fifth Assessment Report, 2014.
- How much will the temperature rise? According to Vostok ice core data, a naive estimate is 10°C for every 100 ppmV. But how long will this take to happen?!

Summary

- Even simple low-dimensional climate models can help explain the mechanisms by which the Earth has transitioned between different states (e.g., from hothouse to snowball). Bifurcation theory is an important tool for the applied mathematician.
- Without greenhouse gases, the Earth would be in a frigid, uninhabitable, snowball state.
- Ice core data provides excellent information about the planet's past climate, revealing a strong correlation between the amount of greenhouse gases in the atmosphere and the temperature.
- Over the past 420,000 years, CO2 concentrations have been in the range 185–310 ppmV. Now they are over 400 ppmV and climbing, resulting in a 1°C rise in temperature over the 20th century mean. We are responsible for this.