MATH 392-02: Seminar in Complex Analytic Dynamics

Homework Assignment #2

DUE DATE: Wed., Feb. 15, start of class.

Homework should be turned in at the BEGINNING OF CLASS. You should write up solutions neatly to all problems, making sure to show all your work. You are encouraged to work on these problems with other classmates, although the solutions you turn in should be your **own** work. Please cite any references (websites, texts, etc.) that you may have used for assistance with the assignment.

Note: Please list the names of any students or faculty who you worked with on the top of the assignment.

1. Do all six parts of Exercise 5 from Ch. 15 of Devaney's text (pp. 218–220). In addition to sketching each orbit, describe it in words as well.

2. Suppose that f and g are analytically conjugate with conjugacy h, that is $h \circ f = g \circ h$, and h is an analytic homeomorphism. Show that h is a conjugacy between f^n and g^n for any $n \in \mathbb{N}$. In other words, show that
$$h \circ f^n \; = \; g^n \circ h \quad \forall n \in \mathbb{N}. \tag{1}$$

3. Use equation (1) to show that, if f and g are analytically conjugate, then h maps period n points of f to period n points of g, and that these points have the same stability types (attracting, repelling, etc.). Specifically, show that z_0 is a period n point for f if and only if $h(z_0)$ is a period n point for g, and that, in this case, $(f^n)'(z_0) = (g^n)'(h(z_0))$.

4. Suppose that f and g are analytically conjugate and that z_0 is an attracting fixed point for f. Prove that if z is asymptotically attracted to z_0 under f, then $h(z)$ is asymptotically attracted to $h(z_0)$ under iteration of g. Conclude that h maps the basin of attraction of z_0 to the basin of attraction of $h(z_0)$. (The *basin of attraction* of a fixed point is the set of all points attracted to the fixed point under iteration.)

5. Find a linear conjugacy $h(z) = \alpha z + \beta$ between the general quadratic family $F_{a,b,d}(z) = az^2 + bz + d$ and the quadratic family $Q_c(z) = z^2 + c$. Give expressions for α and β, and find the equation relating the parameter c to a, b and d. This problem shows that we can restrict our study of the dynamics of quadratic polynomials to the family $z^2 + c$.

6. Show that ∞ is a super-attracting fixed point for *any* polynomial $p(z) = a_d z^d + a_{d-1} z^{d-1} + \cdots + a_1 z + a_0$ of degree $d \geq 2$.

7. For each of the following functions, show that ∞ is a fixed point and determine the type of fixed point (attracting, repelling, neutral, or super-attracting).

   a) $R(z) = \dfrac{2z^3 + 1}{3z^2}$ **b)** $f(z) = ze^{1/z}$ **c)** $g(z) = z^2 e^{1/z}$