MATH 392-02: Seminar in Complex Analytic Dynamics

Homework Assignment #1

DUE DATE: Fri., Feb. 3, start of class.

Homework should be turned in at the BEGINNING OF CLASS. You should write up solutions neatly to all problems, making sure to show all your work. You are encouraged to work on these problems with other classmates, although the solutions you turn in should be your **own** work. Please cite any references (websites, texts, etc.) that you may have used for assistance with the assignment.

Note: Please list the names of any students or faculty who you worked with on the top of the assignment.

1. Suppose that $f(z)$ is a complex polynomial of degree d, with $d \geq 2$. Show that f has $d+1$ fixed points counting multiplicities, including the fixed point at ∞. "Counting multiplicities" means that a fixed point might occur as a repeated root of the fixed point equation, in which case we would count it more than once. *Note:* You should verify that ∞ is a fixed point by showing that $\lim_{z \to \infty} f(z) = \infty$. This can be accomplished using the LIPI theorem from last semester.

2. By way of a counterexample, explain why the case $d = 1$ must be excluded in the statement of the previous problem.

3. Suppose that $f(z)$ is a continuous function and that $\lim_{n \to \infty} f^n(z_0) = p$ for two complex numbers z_0 and p. Show that p is a fixed point of f. It follows that if the orbit of z_0 is asymptotic, then it must approach a fixed point. *Hint:* Go back to your MATH 242 notes and pick a useful definition of continuity. (Make sure you believe it holds in the complex case.)

4. Find the period two-cycle for the function $f(z) = z^2 - i$. Express your answer in rectangular form (i.e., $a + b\,i$). Classify the cycle as attracting, repelling, neutral or super-attracting.

5. Suppose that $f(z)$ is an analytic function and that $z_0, z_1, \ldots, z_{n-1}$ is a period n-cycle. Show that the cycle is super-attracting if and only if at least one of the points z_i on the cycle is a critical point of f.

6. Do the following exercises from Ch. 15 of Devaney's text (pp. 218–220): **8c, 8g, 10, 11, 16**.