Seminar in Complex Analytic Dynamics

MATH 392-02, MWF 11:00 - 11:50, Swords 330, Spring 2012

Professor Gareth Roberts

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Office hours: Mon. 1:00 - 2:00, Tues. 1:00 - 3:00, Wed. 8:00 - 8:50 (in Cool Beans) and 1:00 - 1:50, or by appointment.

Recommended Text: *A First Course In Chaotic Dynamical Systems*, Robert L. Devaney

Course Prerequisites: MATH 305 (Complex Analysis)

Web page: `http://mathcs.holycross.edu/~groberts/Courses/MA392-CD/homepage.html` Homework assignments, project information, schedule changes, useful links and other important information will be posted at this site. Please bookmark it!

Course objectives:

- Study the essential techniques and concepts in complex dynamics.
- Become proficient at making clear and coherent mathematical arguments.
- Learn to read and comprehend advanced mathematical writing.
- Work and communicate with your peers.
- Have FUN learning complex dynamics!

Syllabus: This course focuses on the fascinating properties and figures that arise when a complex analytic function is iterated. For a given complex-valued function $f(z)$, the general idea is to understand the set of orbits

$$\{z_0, f(z_0), f(f(z_0)), f(f(f(z_0))), \ldots\}$$

obtained by repeatedly plugging in a complex number z_0 and its successive images into the function $f(z)$. The orbit will usually vary depending on the initial starting point z_0. For example, z_0 may remain fixed under iteration (e.g., $2, 2, 2, \ldots$), which is known as a *fixed point*. Or, the orbit may bounce back and forth between two particular values (e.g., $-1, i, -1, i, -1, i, \ldots$) in which case it is called a *period-two cycle*. Another possibility is that the orbit limits on some complex number under repeated iteration. Of particular interest is whether a given cycle or fixed point attracts or repells nearby orbits. Here, properties of the derivative and analytic functions become important.

The dynamics of a complex function separate into two contrasting sets, the *Julia set*, where the "chaotic" behavior occurs, and the *Fatou set*, where the "tame" dynamics takes place. These sets are named after two of the primary founders of the subject of complex dynamics, the French mathematicians Gaston Julia and Pierre Fatou. The Julia set is typically an intricate and often mesmerizing fractal, that is, a self-similar set where enlargements of small pieces of the set resemble the original set. If you've seen pictures of fractals, then you've likely been looking at Julia sets, or more specifically *filled Julia sets*. We will explore the mathematics behind these two sets, building on some of the material from complex analysis as well as the theory of dynamical systems.

The Julia and Fatou sets lie in the dynamical plane, where the orbits live. However, it is quite useful and fascinating to study a family of functions in terms of a complex parameter. One example we will study in great detail is $Q_c(z) = z^2 + c$ where $c \in \mathbb{C}$ is treated as a parameter. For each choice of c, we obtain a specific dynamical system with a Julia set and a Fatou set. If we then study all values of c for which the corresponding Julia set of Q_c is connected, we obtain a truly marvelous set in the parameter plane called the *Mandelbrot set*, named for the groundbreaking computer graphics work of Benoit Mandelbrot. We will investigate some of the remarkable topological and number theoretical properties of the Mandelbrot set.

We will cover much of the material in Chapters 15, 16 and 17 of Devaney's textbook and also work our way through parts of the nice expository paper "Complex Analytic Dynamics on the Riemann Sphere" (*Bulletin of the American Mathematical Society*, **11**, no. 1, 1984) by Paul Blanchard. The computer will be a useful tool in our understanding of the dynamics and structure of the Julia set and Mandelbrot set, and a few computer labs will be assigned to help develop that understanding.

A rough outline of topics for the semester is given below, although the order these topics are covered may vary. Some topics will be approached at a very advanced level.

- Iteration and types of orbits: fixed points, periodic points and their stability type (attracting, repelling or neutral), dense orbits
- Basic dynamical systems theory: conjugacy, linear maps, attracting and repelling fixed point theorems, chaotic maps
- The Julia set and the Fatou set: examples, definition, properties of, normal families, Montel's theorem, the fundamental decomposition theorem, dynamics in the Fatou set (Sullivan domains)
- The dynamics of $Q_c(z) = z^2 + c$: filled Julia sets, Cantor sets, supersensitivity, fractals, bifurcations
- The Mandelbrot set: definition, properties of, the fundamental dichotomy, fixed point and period-two regions, periods of bulbs, Farey addition, external rays, open questions
- Other topics to be considered (likely as final projects): Newton's method as a complex dynamical system, tricorns, iteration of cubic polynomials, dynamics of λe^z, dynamics of trig functions, Sierpinski carpets and gaskets as Julia sets

Homework: There will be regular homework assignments throughout the semester. Assignments will be posted on the course web page. There will also be a few computer labs that require the use of special software available on the Internet. While you are allowed and encouraged to work on homework problems with your classmates, the solutions you turn in to be graded should be your own. Take care to do your final write-up of homework problems on your own and **in your own words**. Plagiarism will not be tolerated and will be treated as a violation of both the departmental policy on academic integrity and the college's policy on academic honesty.

NOTE: Late homework will not be accepted. However, you will be allowed ONE "mulligan" over the course of the semester where you can turn in the assignment up to one week after the original due date.

Midterm Exam: There will be one midterm exam on the evening of **Wednesday, February 29** from 7:00 - 8:30 pm. Any conflicts with the exam must be legitimate and brought to my attention well before the exam is scheduled. If you have any specific learning disabilities or special needs and require accommodations, please let me know early in the semester so that your learning needs may be appropriately met. You will need to contact the director of Disability Services (Hogan 215, x3693) to obtain documentation of your disability.

In-class Presentations: During the semester you will be expected to give two in-class presentations, one covering some particular topic related to that days' lecture and the other (on a different day) consisting of the solution to a particular homework problem (likely on the day the homework is due.) These presentations will be brief and are designed to enhance your oral communication skills as well as prepare you for your final project presentation. You are encouraged to consult with me before each of your presentations.

Final Project: You are required to complete a substantial final project focusing on some particular aspect or application related to the course material. Details and suggestions of topics will be distributed later in the semester. Your project will include both a written report and an in-class presentation during the final week of class. You will be allowed to work in small groups (2-3 people) for the project although it is expected that all members of a given group will contribute equally.

Academic Integrity: The Department of Mathematics and Computer Science has drafted a policy on academic integrity to precisely state our expectations of both students and faculty with regards to cheating, plagiarism, academic honesty, etc. You are required to read this policy and sign a pledge agreeing to uphold it. A violation of the Departmental Policy on Academic Integrity will result in a 0 for that assignment (or exam) and a letter describing the occurrence of academic dishonesty will be sent to your Class Dean.

Grade: Your course grade will be based on the scores you receive for each of the following items:

- classroom participation/interest 5%
- in-class presentations 10%
- midterm exam 20%
- homework and labs 35%
- final project/presentation 30%

How to do well in this course:

- Attend class, participate and ask questions.
- Work with your classmates. Organize study groups.
- Be an active, responsible learner.
- Read the text and course handouts carefully.