Dynamical Systems, MATH 374

Homework Assignment #4

Due Fri., Oct. 20, 5:00 pm

You should write up solutions neatly to all problems, making sure to show all your work. A nonempty subset will be graded. You are encouraged to work on these problems with other classmates, and it is ok to use internet sources for help if it's absolutely necessary (with proper citation); however, the solutions you turn in should be YOUR OWN WORK and written in YOUR OWN WORDS.

Note: Please list the names of any students or faculty who you worked with on the assignment.

A Geometry of Nature

Read *A Geometry of Nature*, the fourth chapter of Gleick's book. Describe the mathematical training of Benoit Mandelbrot. Why did he characterize himself as a "nomad-by-choice"? How was he viewed by other mathematicians and why? What was his geometric approach to understanding natural phenomena? We will explore some of the material here in Chapter 14 of Devaney's text.

Problems on Topological Conjugacy

1. Suppose that f and g are topologically conjugate with conjugacy h, that is $h \circ f = g \circ h$. Show that h is a conjugacy between f^n and g^n for any $n \in \mathbb{N}$. In other words, show that

$$h \circ f^n \;=\; g^n \circ h \quad \forall n \in \mathbb{N}. \tag{1}$$

   Hint: Use the fact that h is invertible.

2. Use equation (1) to show that, if f and g are topologically conjugate, then h maps period n points of f to period n points of g. Specifically, show that p is a period n point for f if and only if $h(p)$ is a period n point for g.

3. Suppose that f and g are topologically conjugate and that p is an attracting fixed point for f. Prove that if x is asymptotically attracted to p under f, then $h(x)$ is asymptotically attracted to $h(p)$ under iteration of g. Conclude that h maps the basin of attraction of p to the basin of attraction of $h(p)$. (The *basin of attraction* of a fixed point is the set of all points attracted to the fixed point under iteration.)

4. Find a linear conjugacy $h(x) = \alpha x + \beta$ between the quadratic map $Q_c(x) = x^2 + c$ and the logistic map $F_\lambda(x) = \lambda x(1-x)$. Give the expressions for α and β and the equation relating the parameters c and λ.

5. Using your conjugacy from the previous question, for what values of λ should we expect to see period-doubling bifurcations occur for the logistic map.

6. Find two dynamical systems f and g such that f and g each have one fixed point p_1 and p_2, respectively, with $f'(p_1) = g'(p_2)$, but f and g are **not** topologically conjugate.

Problems on Symbolic Dynamics

Chapter 9 Exercises (Devaney) (pp. 111 - 113)

Problems: 1, 2, 3, 7, 8, 18a, 18b, 18f

Hint: For #18, use the Proximity Theorem. Given ϵ, the goal is to find a δ that insures that $F(s)$ and $F(t)$ are within ϵ of each other, whenever s and t are within δ of each other. Try to choose the optimal δ in each proof.