

12. How many fixed points does σ_N have? How many 2-cycles? How many cycles of prime period 2?

Let σ_N be the shift map on Σ_N . Specifically, let $\sigma_N: \Sigma_N \rightarrow \Sigma_N$ with

$$\sigma_N(s_0 s_1 s_2 \dots) = (s_1 s_2 s_3 \dots).$$

Now σ_N has N fixed points; indeed,

$$\text{fix } \sigma_N = \{(000\dots), (111\dots), \dots, (kkk\dots)\}$$

where $k = N - 1$. Recall that σ_2 has two points of prime period 2 and we wonder if σ_N has N points of prime period 2. It turns out that this is *not* the case since any sequence of the form $(\overline{s_0 s_1})$ is of period 2, and there are N^2 such points. But N of these are fixed, and so there are $N^2 - N = N(N - 1)$ points of prime period 2.