6. Give an example of a sequence midway between $(000\ldots)$ and $(111\ldots)$. Give a second such example. Are there any other such points? Why or why not?

One of the two points midway between $(\overline{0})$ and $(\overline{1})$ is $(0\overline{1})$ since $d[(0\overline{1}),(\overline{0})] = d[(0\overline{1}),(\overline{1})] = 1$. The other point is $(1\overline{0})$, and these are the only such symbol sequences by virtue of the Proximity Theorem.

7. Let $M_{01} = \{\mathbf{s} \in \Sigma \mid s_0 = 0, s_1 = 1\}$ and $M_{101} = \{\mathbf{s} \in \Sigma \mid s_0 = 1, s_1 = 0, s_2 = 1\}$. What is the minimum distance between a point in M_{01} and a point in M_{101}? Give an example of two sequences that are this close to each other.

Let $\mathbf{s} \in M_{01}$ and $\mathbf{t} \in M_{101}$. Then $d[\mathbf{s},\mathbf{t}] \geq 3/2$ since $\mathbf{s}$ and $\mathbf{t}$ differ in the first two positions. For example, let $\mathbf{s} = (01\overline{1})$ and $\mathbf{t} = (10\overline{1})$. Then $d[\mathbf{s},\mathbf{t}] = 3/2$. More generally, suppose $\mathbf{t} = (101t_3t_4\ldots)$. Then $\mathbf{s} = (011t_3t_4\ldots)$ is exactly $3/2$ units away from $\mathbf{t}$.

8. What is the maximum distance between a point in M_{01} and a point in M_{101}? Give an example of two sequences that are this far apart.

The maximum distance is 2 units since the strings need not agree at any position. Given any $\mathbf{t} \in M_{101}$, all strings in M_{01} of the form $(010\hat{t}_3\hat{t}_4\ldots)$ are 2 units away from $\mathbf{t}$.

The N-Shift:

The following seven exercises deal with the analogue of the shift map and sequence space for sequences that have more than two possible entries, the space of sequences of N symbols.

$$
\begin{aligned}
d_N[\mathbf{s},\mathbf{t}] + d_N[\mathbf{t},\mathbf{u}] &= \sum_{i=0}^{\infty} \frac{|s_i - t_i|}{N^i} + \sum_{i=0}^{\infty} \frac{|t_i - u_i|}{N^i} \\
&= \sum_{i=0}^{\infty} \frac{|s_i - t_i| + |t_i - u_i|}{N^i} \\
&\geq \sum_{i=0}^{\infty} \frac{|s_i - u_i|}{N^i} \\
&= d_N[\mathbf{s},\mathbf{u}].
\end{aligned}
$$

Thus d_N is a metric and $\langle d_N, \Sigma_N\rangle$ is a metric space.

11. What is the maximal distance between a pair of sequences in Σ_N?

The maximum value of $|s_i - t_i|$ is $N-1$, and therefore, the maximum distance between two sequences in Σ_N is

$$
\begin{aligned}
d_N[\mathbf{s},\mathbf{t}] &= \sum_{i=0}^{\infty} \frac{|s_i - t_i|}{N^i} \\
&\leq \sum_{i=0}^{\infty} \frac{N-1}{N^i} \\
&= (N-1)\frac{1}{1-1/N} \\
&= N.
\end{aligned}
$$

So, in general, the maximum distance can be no more than the size of the alphabet.