2. S_2 is the set of all rationals in $[0,1]$ of the form $p/2^n$ where p and n are natural numbers.

Note that S_2 contains all multiples of 1/2, all multiples of 1/4, etc. In fact, $x \in S_2$ if and only if x has a terminating binary expansion. Assuming this to be true for the moment, we may easily show that S_2 is dense in $[0,1]$ as follows:

Let $w = 0.b_1b_2b_3\ldots$ be an arbitrary point in $[0,1]$. If w terminates, then we are done, so suppose it does not. Then the sequence

$$0.b_1,\ 0.b_1b_2,\ 0.b_1b_2b_3,\ldots$$

obviously converges to w, and each element of this sequence is in S_2. q.e.d.

Using a bisection technique, we now exhibit such a sequence in S_2 converging to w. Take the unit interval, divide it in half, and determine which half contains w. Discard the half which does not. Halve the remaining interval, and again ask which half contains w. Continue this halving process, each time throwing away the half interval which does not contain w. This binary search technique, as it's called, captures w to any degree of precision one cares to specify.

A corresponding algorithm is given in Figure 10.1. The variables l_k and r_k are the left-hand and right-hand endpoints of the subintervals, respectively,

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[1] We say that w is a globally attracting fixed point for F_w.