weakly attracting (resp. weakly repelling) on the right and weakly repelling (resp. weakly attracting) on the left.

Case 2: *Suppose $F''(p) = 0$. If $F'''(p) < 0$ (resp. $F'''(p) > 0$), then p is weakly attracting (resp. weakly repelling).*

But what if $F'(p) = -1$? As we saw in Exercises 4h–4k, the trick is to apply Theorem 5.2 to F^2. First of all, by Proposition 5.1, a neutral fixed point for F is also a neutral fixed point for F^2, but with $(F^2)'(p) = 1$. Furthermore, using the chain and product rules for derivatives, we find that

$$(F^2)''(x) = F''(F(x)) \cdot [F'(x)]^2 + F'(F(x)) \cdot F''(x).$$

Evaluating this derivative at p, we obtain

$$\begin{aligned}(F^2)''(p) &= F''(F(p)) \cdot [F'(p)]^2 + F'(F(p)) \cdot F''(p)\\ &= F''(p) - F''(p)\\ &= 0\end{aligned}$$

and so case 2 of Theorem 5.2 applies. What remains is the somewhat tedious computation of $(F^2)'''(p)$. The more industrious reader will be inclined to verify that

$$(F^2)'''(x) =$$
$$F'''(F(x)) \cdot [F'(x)]^3 + 3F''(F(x)) \cdot F'(x) \cdot F''(x) + F'(F(x)) \cdot F'''(x)$$

from which it follows that

$$(F^2)'''(p) = -2F'''(p) - 3[F''(p)]^2.$$

When this quantity is negative,[8] the neutral fixed point p is weakly attracting for F^2, and hence for F.

Let's summarize this result in the following

Theorem 5.3 *Let p be a neutral fixed point for F with $F'(p) = -1$. If the quantity $-2F'''(p) - 3[F''(p)]^2$ is negative (resp. positive), then p is weakly attracting (resp. weakly repelling).*

[8] Which is the same as saying F has negative Schwarzian derivative at p. (See Chapter 12.)

Unfortunately, there are still cases of neutral fixed points that have yet to be considered. What if $F'(p) = 1$ and $F''(p) = F'''(p) = 0$, for instance? Under these conditions, Theorem 5.2 does not apply and we must continue evaluating higher derivatives at p until one of them is nonzero. Even this will not always work, however, since there are maps having a neutral fixed point satisfying $F'(p) = 1$, and for which $F^{(n)}(p) = 0$ for all $n > 1$. The identity map is one such (trivial) example—can you find others?