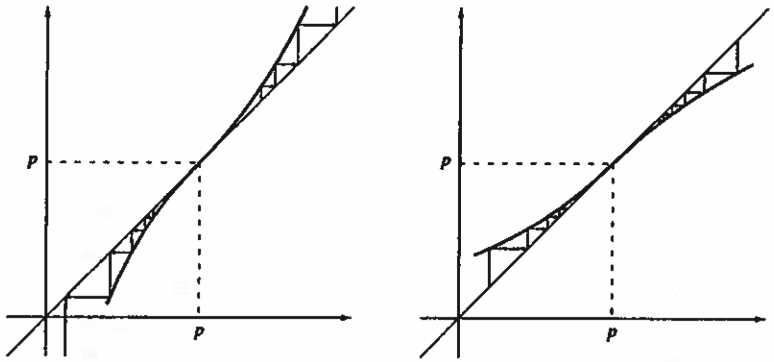


(a) $F(p) = p, F'(p) = 1, F''(p) = 0$, and $F'''(p) > 0$ (b) $F(p) = p, F'(p) = 1, F''(p) = 0$, and $F'''(p) < 0$

Figure 5.8: Two more cases of a neutral fixed point.

p is weakly repelling in this case. But F'' is increasing in a neighborhood of p since it's negative to the left and positive to the right. Therefore, p is weakly repelling provided the derivative of F'' is positive at p, that is, if $F'''(p) > 0$.

8. Repeat Exercise 7, but this time assume that $F'''(p) < 0$. Show that p is weakly attracting.

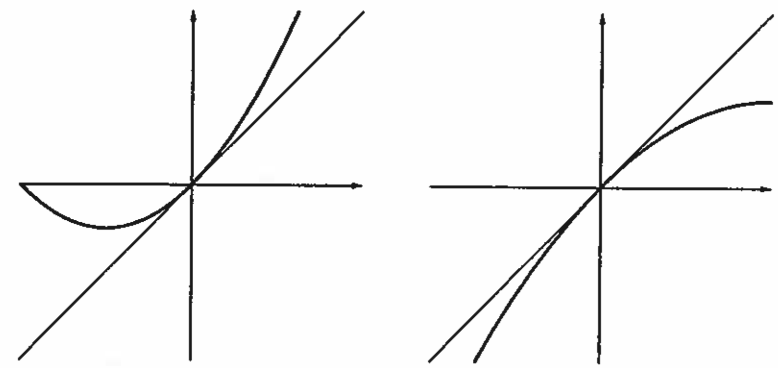
In this case, we suppose that $F''(x)$ is positive to the left of p, and negative to the right of p, so that the graph of F is concave up to the left and concave down to the right (see Figure 5.8b). Arguing as above, it follows that $F'''(p) < 0$.

9. Combine the results of Exercises 5–8 to state a **Neutral Fixed Point Theorem.**

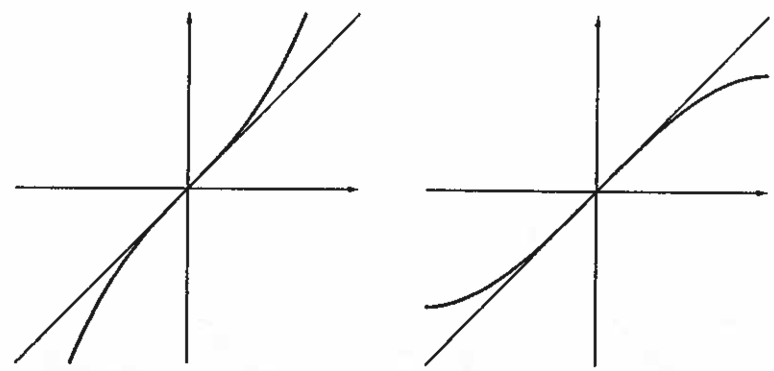
The four basic cases are illustrated in Figure 5.9 for $p = 0$ and summarized below.

Theorem 5.2 *Let p be a neutral fixed point for F with $F'(p) = 1$.*

Case 1: *Suppose $F''(p) \neq 0$. If $F''(p) < 0$ (resp. $F''(p) > 0$), then p is*



(a) $F(0) = 0, F'(0) = 1, F''(0) > 0$ (b) $F(0) = 0, F'(0) = 1, F''(0) < 0$



(c) $F(0) = 0, F'(0) = 1, F''(0) = 0$, and $F'''(0) > 0$ (d) $F(0) = 0, F'(0) = 1, F''(0) = 0$, and $F'''(0) < 0$

Figure 5.9: The Four Canonical Forms of Neutral Fixed Points.