repelling? From the graph depicted in Figure 5.5b, it appears that $\tan x > x$ for $0 < x < \pi/2$ (see the derivation preceding Exercise 4d) and that $\tan x < x$ for $-\pi/2 < x < 0$, and so the origin must be weakly repelling. In fact, it can be shown that *all* of T's fixed points are repelling, and the reader is invited to experiment with this interesting map on the computer.

The results of Exercise 7 suggest an alternative approach to this problem. The reader may verify that $T'(x) = \sec^2 x$, $T''(x) = 2\tan x \sec^2 x$, and $T'''(x) = 2\sec^2 x(\tan^2 x + \sec^2 x)$. Evaluating these derivatives at the fixed point, we find that $T'(0) = 1$, $T''(0) = 0$, and $T'''(0) = 2 > 0$. The origin is therefore weakly repelling by Exercise 7.

4f) $F(x) = x + x^3$ (see Figure 5.5c)

This is the canonical example of a map with a weakly repelling fixed point. First of all, note that 0 is fixed by F. Indeed,

$$\text{fix } F = \{0\}.$$

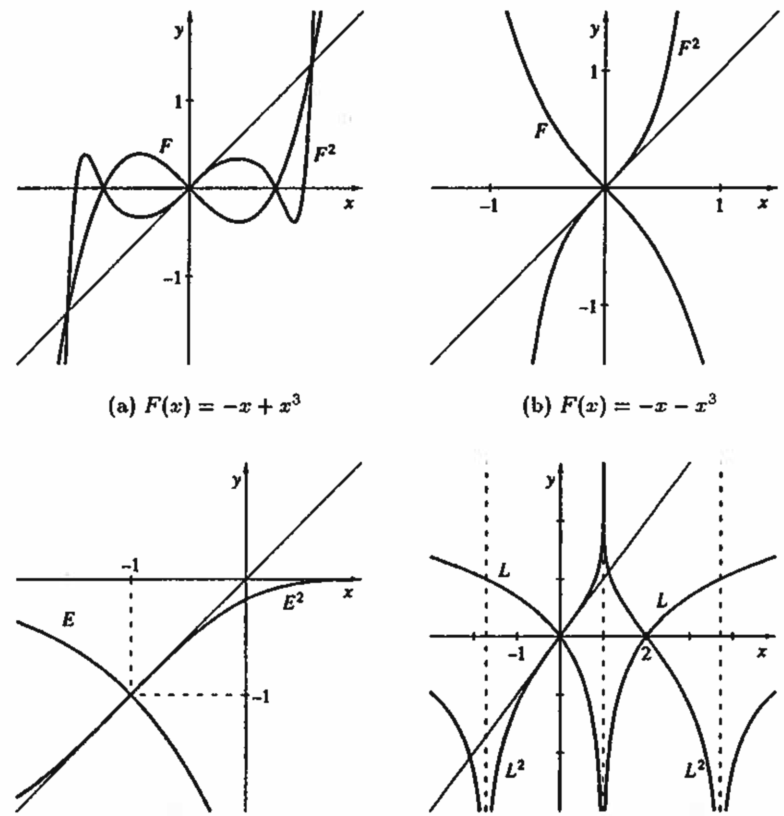
Since $F'(x) = 1 + 3x^2$, we see that $F'(0) = 1$ and so 0 is neutral. The fact that 0 is weakly repelling follows from graphical analysis and the fact that the graph of F lies below the diagonal for negative x and above the diagonal for positive x. This observation is verified using the results of Exercise 7; that is, 0 is weakly repelling since $F''(0) = 0$ and $F'''(0) = 6 > 0$.

We remark that the graph of F has the same basic shape as the graph in Exercise 4e.

4g) $F(x) = x - x^3$ (see Figure 5.5d)

Likewise, this is the canonical example of a map with a weakly attracting fixed point. Again, 0 is the only fixed point of F with $F'(0) = 1$ and $F''(0) = 0$. But this time the graph of F lies *above* the diagonal to the left of the origin and *below* the diagonal to the right. This is because $F'''(0) = -6 < 0$ (see Exercise 4d for a similar situation) and graphical analysis confirms that the origin is weakly attracting in this case. Note that F has local extrema at $x = \pm 1$ (verify this) which do not effect the local dynamics about the fixed point.

4h) $F(x) = -x + x^3$



(a) $F(x) = -x + x^3$ (b) $F(x) = -x - x^3$

(c) $E(x) = -ee^x$ (d) $L(x) = \ln|x - 1|$

Figure 5.6: Neutral fixed points with a derivative of -1.