(5.4), we have

$$\frac{\sin\theta}{2} < \frac{\theta}{2}$$

which implies that

$$\sin\theta < \theta.$$

This result holds for all θ in the first quadrant, and may even be extended to arbitrary $\theta > 0$. (Exercise: Show $\sin\theta < \theta$ for $\pi/2 < \theta < \pi$, for instance.)

At this point, we've shown half of inequality (5.1). For the rest, observe that the area of $\Delta OP'Q$ is

$$\frac{\tan\theta}{2}$$

since the base of the triangle has length 1. This area clearly exceeds that of the wedge, and so

$$\frac{\tan\theta}{2} > \frac{\theta}{2}$$

which implies that

$$\tan\theta > \theta.$$

We remark that this result can *not* be extended to arbitrary $\theta > 0$. This completes the detour.

4d) $S(x) = \sin x$

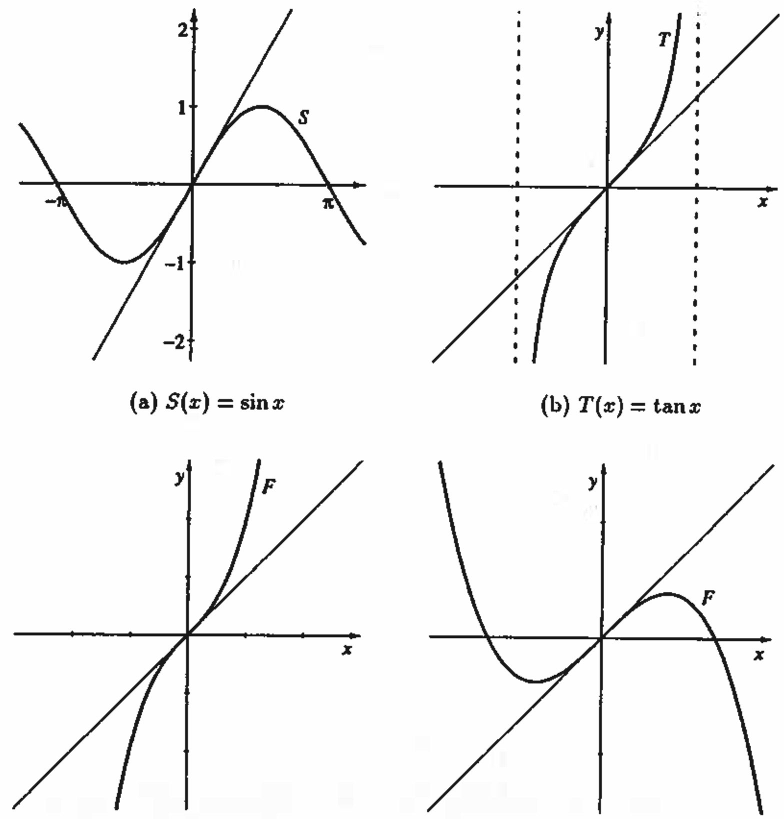
From the graph of S (see Figure 5.5a), we see that $\text{fix}\, S = \{0\}$. But once again it's difficult (if not impossible!) to show this by solving the equation $S(x) = x$ for x. At any rate, we see from the graph of S that $\sin x < x$ for all $x > 0$ (see preceding derivation) and that $\sin x > x$ for all $x < 0$. Thus the origin is attracting, but only weakly so since $S'(0) = \cos 0 = 1$.

Anticipating Exercise 8, we illustrate an alternative, more mechanical approach to this problem. Observe that $S'(x) = \cos x$, $S''(x) = -S(x)$, and $S'''(x) = -S'(x)$. Thus, $S'(0) = 1$, $S''(0) = 0$, and $S'''(0) = -1 < 0$. By Exercise 8, we conclude that the origin is weakly attracting.

4e) $T(x) = \tan x$

We know that $0 \in \text{fix}\, T$ since $\tan 0 = 0$,[6] and this fixed point is indeed neutral since $T'(0) = \sec^2 0 = 1$. But is it weakly attracting or weakly

[6] Observe that T has an infinite number of fixed points since it's π-periodic.



(a) $S(x) = \sin x$ (b) $T(x) = \tan x$

(c) $F(x) = x + x^3$ (d) $F(x) = x - x^3$

Figure 5.5: Examples of neutral fixed points which are also inflection points. Such points are either weakly repelling or weakly attracting.