The number ϕ is called the **golden ratio**[1] and arises naturally in connection with the ubiquitous Fibonacci sequence.[2] Now, since

$$F'(x) = 4x^3 - 8x = 4x \cdot Q(x),$$

we have that $F'(-1) = 4$ and $F'(2) = 16$, and hence, both integral fixed points are repelling. But what about $-\phi$ and $1/\phi$? Recall that a fixed point of Q is also a fixed point for F, and since Q is an even function, we may compute $Q(\phi)$ instead of $Q(-\phi)$. That is,

$$\begin{aligned}\phi^2 - 2 &= \left(\frac{1+\sqrt{5}}{2}\right)^2 - 2\\ &= \frac{6+2\sqrt{5}}{4} - \frac{8}{4}\\ &= \frac{-2+2\sqrt{5}}{4}\\ &= \frac{-1+\sqrt{5}}{2}\\ &= \frac{1}{\phi}.\end{aligned}$$

Similarly, we find that

$$\frac{1}{\phi^2} - 2 = -\phi,$$

and hence, $-\phi$ and $1/\phi$ constitute a 2-cycle for Q (see Exercise 3.3). And since

$$F'(-\phi) = -4\phi(\phi^2 - 2) = -4\phi\frac{1}{\phi} = -4$$

along with

$$F'\left(\frac{1}{\phi}\right) = \frac{4}{\phi}\left(\frac{1}{\phi^2} - 2\right) = \frac{4}{\phi}(-\phi) = -4,$$

$-\phi$ and $1/\phi$ are repelling fixed points for F.

[1] Some authors call ϕ the golden section while still others define it as $(\sqrt{5}-1)/2$. The latter and $(\sqrt{5}+1)/2$ are reciprocals of one another, and a single unit apart on the real line.

[2] For a particularly lucid introduction to the Fibonacci numbers, see chapter 11 in: Ogilvy, C. Stanley and John T. Anderson. *Excursions in number theory*. New York:

1f) $S(x) = \frac{\pi}{2}\sin x$

$\operatorname{fix} S = \{0, \pm\pi/2\}$.

$S'(x) = \frac{\pi}{2}\cos x$.

$S'(0) = \pi/2 > 1 \Rightarrow 0$ is repelling.

$S'(\pm\pi/2) = (\pi/2)\cos(\pm\pi/2) = 0 \Rightarrow \pm\pi/2$ are superattracting.

1g) $S(x) = -\sin x$

$\operatorname{fix} S = \{0\}$.

$S'(x) = -\cos x$.

$S'(0) = -1$, and so -1 is neutral.

1h) $F(x) = x^3 - x$

$$\begin{aligned}x^3 - x = x &\Rightarrow x^3 - 2x = 0\\ &\Rightarrow x(x^2-2) = 0\\ &\Rightarrow x = 0 \quad\text{or}\quad x = \pm\sqrt{2}\end{aligned}$$

Therefore, $\operatorname{fix} F = \{0, \pm\sqrt{2}\}$.

$F'(x) = 3x^2 - 1$.

$F'(0) = -1 \Rightarrow 0$ is neutral.

$F'(\pm\sqrt{2}) = 3(\pm\sqrt{2})^2 - 1 = 5 \Rightarrow \pm\sqrt{2}$ are repelling.

1i) $A(x) = \arctan x$

$\operatorname{fix} A = \{0\}$.

$A'(x) = 1/(1+x^2)$.

$A'(0) = 1 \Rightarrow 0$ is neutral.

1j) $T(x) = \begin{cases} 2x & \text{if } x \le 1/2\\ 2-2x & \text{if } x > 1/2\end{cases}$

Setting each piece of this two part function equal to x yields $\operatorname{fix} T = \{0, 2/3\}$. Also,

$$T'(x) = \begin{cases} 2 & \text{if } x < 1/2\\ -2 & \text{if } x > 1/2\end{cases}$$

since T is piecewise linear, but the derivative of T is not defined at $x =$