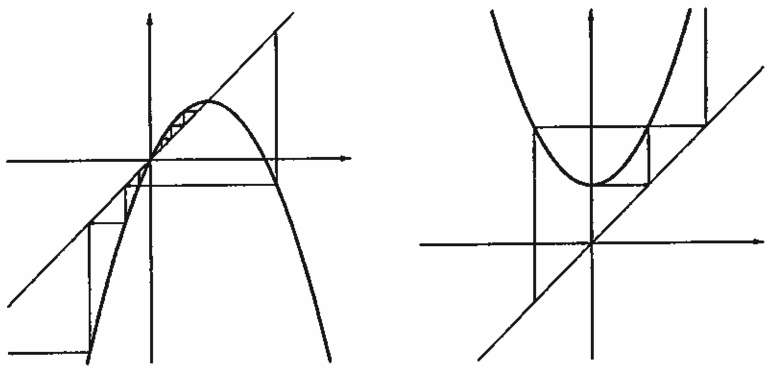

(a) The graph of $F(x) = 2x(1-x)$. (b) The graph of $F(x) = x^2 + 1$.

Figure 4.6: The dynamics of two more quadratic functions.

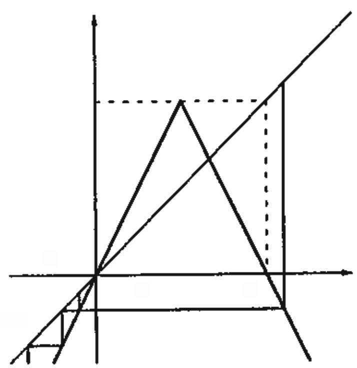

Figure 4.7: The dynamics of the tent map $T(x) = 1 - |2x - 1|$.

See Figure 4.8.

4. Perform a complete orbit analysis for each of the following functions.

4a) $F(x) = \frac{1}{2}x - 2$

First, let's find a fixed point for F:

$$\begin{aligned} \frac{1}{2}x - 2 &= x \\ \Rightarrow\ -2 &= \frac{1}{2}x \\ \Rightarrow\ -4 &= x. \end{aligned}$$

Thus, $\text{fix}\, F = \{-4\}$, and it so happens that $F^n(x) \to -4$ as $n \to \infty$ for *all* x. See Figure 4.9.

4b) $A(x) = |x|$

This exercise was essentially solved at the end of Chapter 3 (see Exercises 6, 7e, and 8) where it was shown that

$$\text{fix}\, A = \{\, x \mid x \geq 0 \,\},$$

and that all other points are eventually fixed after just one iteration.

4c) $F(x) = -x^2$

From the graph of F in Figure 4.10, we see that $\text{fix}\, F = \{-1, 0\}$. Since $F(1) = -1$, we also see that $x = 1$ is eventually fixed. For $-1 < x < 1$, $F^n(x) \to 0$ as $n \to \infty$, and for $|x| > 1$, $F^n(x) \to -\infty$ as $n \to \infty$. That is, 0 is attracting and -1 is repelling.

4d) $F(x) = -x^5$

First, let's compute the fixed points of F:

$$\begin{aligned} -x^5 &= x \\ \Rightarrow\ 0 &= x + x^5 \\ \Rightarrow\ 0 &= x(1 + x^4). \end{aligned}$$

Therefore, $0 \in \text{fix}\, F$ and the other four fixed points are complex. Now,

$$F'(x) = -5x^4$$