Dynamical Systems, MATH 374

Homework Assignment #4

Due Thurs., March 11, Start of class

You should write up solutions neatly to all problems, making sure to show all your work. A nonempty subset will be graded. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your own work.

Note: Please list the names of any students or faculty who you worked with on the assignment.

A Geometry of Nature

Read *A Geometry of Nature*, the fourth chapter of Gleick's book. Describe the mathematical training of Benoit Mandelbrot. Why did he characterize himself as a "nomad-by-choice"? How was he viewed by other mathematicians and why? What was his geometric approach to understanding natural phenomena? We will explore some of the material here in Chapter 14 of Devaney's text.

Problems on Topological Conjugacy

1. Suppose that f and g are topologically conjugate with conjugacy h, that is $h \circ f = g \circ h$. Show that h is a conjugacy between f^n and g^n for any $n \in \mathbb{N}$. In other words, show that

$$h \circ f^n = g^n \circ h \quad \forall n \in \mathbb{N}. \tag{1}$$

   Hint: Use the fact that h is invertible.

2. Use equation (1) to show that, if f and g are topologically conjugate, then h maps period n points of f to period n points of g. Specifically, show that p is a period n point for f if and only if $h(p)$ is a period n point for g.

3. Suppose that f and g are topologically conjugate and that p is an attracting fixed point for f. Prove that if x is asymptotically attracted to p under f, then $h(x)$ is asymptotically attracted to $h(p)$ under iteration of g. Conclude that h maps the basin of attraction of p to the basin of attraction of $h(p)$. (The *basin of attraction* of a fixed point is the set of all points attracted to the fixed point under iteration.)

4. Find a linear conjugacy $h(x) = \alpha x + \beta$ between the quadratic map $Q_c(x) = x^2 + c$ and the logistic map $F_\lambda(x) = \lambda x(1 - x)$. Give the expressions for α and β and the equation relating the parameters c and λ.

5. Using your conjugacy from the previous question, for what values of λ should we expect to see period-doubling bifurcations occur for the logistic map.

6. Find two dynamical systems f and g such that f and g each have one fixed point p_1 and p_2 (respectively) with $f'(p_1) = g'(p_2)$ but f and g are **not** topologically conjugate. (Courtesy of *Matt Brady*.)

Problems on Symbolic Dynamics

Chapter 9 Exercises (Devaney) (pp. 111 - 113)

Problems: 1, 2, 7, 8, 18a, 18b, 18f

Note: For the continuity problems in #18, given ϵ, the goal is to find a δ that works in the definition of continuity or convince yourself no such δ exists. For starters, go over the proof from class (3/9) that the shift map is continuous.