Dynamical Systems

MATH 374, TuTh 11:00 - 12:15, Swords 328, Spring 2010

Dr. Gareth Roberts

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Office hours: Tues. 2 - 3:30, Wed. 1 - 3, Thur. 1 - 2 or by appointment.

Required Texts: *A First Course In Chaotic Dynamical Systems*, Robert L. Devaney and *Chaos: Making a New Science* (20th-anniversary edition), James Gleick

Course Prerequisites: MATH 242

Web page: `http://mathcs.holycross.edu/~groberts/Courses/MA374/homepage.html`
Homework assignments, computer projects, exam materials, schedule changes, useful links and other important information will be posted at this site. Please bookmark it!

Course objectives:

- Learn, apply and synthesize the mathematical techniques of dynamical systems.
- Become proficient at making clear and coherent mathematical arguments.
- Work and communicate with your peers.
- Have fun learning dynamical systems!

Syllabus: The goal of this course is to describe some of the fascinating ideas and applications in the field of dynamical systems. Any process that changes over time can be viewed as a dynamical system. The weather, the Dow Jones Industrial Average, the amount of money in your bank account, the population of north Atlantic right whales and the motion of the planets are all examples of processes that can be modeled as dynamical systems.

We will restrict our attention to discrete dynamical systems as opposed to continuous ones (modeled by differential equations), the key concept being the **iteration** of a function. Specifically, given a function $f : X \mapsto X$ mapping some space X to itself and an initial seed x_0 in X, we are interested in following the **orbit** of x_0 under iteration of f. The orbit of x_0 is the sequence of numbers

$$x_0,\ f(x_0),\ f(f(x_0)),\ f(f(f(x_0))), \ldots$$

For a simple example, typing a number and repeatedly hitting the sine key on a calculator gives the orbit of that number under the sine function.

There are two overarching themes that will guide our study. The first aim is to classify the underlying dynamical behavior of a given system. What types of orbits exist? Are there periodic orbits? What are they and how many? Are there "dense" orbits? Is it possible to describe the fate of all orbits? What is the structure of the set of points where the "interesting" dynamical behavior occurs? The second goal is to consider a family of functions depending on a parameter and study the changes in dynamical behavior as the parameter varies. This has particular relevance to real-world systems since measurements in the field are always approximations. What dynamical structures persist? What are the key values where the dynamical behavior changes substantially?

By what mechanisms do these changes occur? For these types of questions we will investigate some very famous bifurcation diagrams.

Our study of dynamical systems will be approached from an analytic, geometric and numerical viewpoint, with the numerical work being conducted through various computer projects. We will cover most of the material in the text except for Chapters 13 and 18. At some point during the semester Professor Devaney will be on campus to give a Departmental Colloquium. Be sure you don't miss his talk!

A rough outline of the semester is as follows:

- Sample Dynamical Systems and Iteration (1 week)
- Orbits: fixed points, periodic points, dense orbits, attractors and repellers (1 1/2 weeks)
- Visualizing Orbits: graphical analysis, histograms (1 week)
- Bifurcations: saddle-node, period doubling, orbit diagrams (1 week)
- Exam I (Chapters 1 through 6)
- Conjugacy: symbolic dynamics, shift map (2 weeks)
- Chaos: chaotic dynamical systems, Sarkovskii's theorem, the Schwarzian derivative (2 weeks)
- Exam II (Chapters 7 through 11)
- Fractals: Sierpinski triangle, dimension, iterated function systems (1 week)
- Complex Dynamical Systems: the Julia set, the Mandelbrot Set (1 1/2 weeks)
- Final Project Presentations (1 week)
- Final Exam (Cumulative)

Homework: There will be approximately 7 - 8 homework assignments given out during the course of the semester. Assignments will be posted on the course web page. There will be a list of problems for you to hand in, a nonempty subset of which will be graded. While you are allowed and encouraged to work on homework problems with your classmates, the solutions you turn in to be graded should be your own. Take care to write up solutions **in your own words**. Plagiarism will not be tolerated and will be treated as a violation of both the departmental policy on academic integrity and the college's policy on academic honesty.

NOTE: LATE homework will NOT be accepted. However, you will be allowed ONE "mulligan" over the course of the semester where you can turn in the assignment up to one week after the original due date.

Computer Projects: There will be three to four computer projects assigned over the course of the semester using software that is available on the web at `http://math.bu.edu/DYSYS/applets/` This website contains java applets from the Dynamical Systems and Technology Project at Boston University overseen by the author of the primary course text, Robert Devaney. The projects are numerical "experiments" designed to discover or reinforce important concepts in the theory of dynamical systems. One project will be a competition to see who can compute the most digits of a famous constant. Projects will be carried out in groups of 2 to 3 people with one typed set of solutions to be turned in for the whole group.

Final Project: You are required to complete a final project focusing on some particular aspect or application related to the course material. Details and suggestions of topics will be distributed later in the semester. You may find a topic of interest as you read James Gleick's book on Chaos. Your project will include both a written report and an in-class presentation during the final week of class. You will be allowed to work in small groups for the project although it is expected that each member of the group will contribute significantly.

Exams: There will be two midterm exams (in class) and a comprehensive final at the end of the semester. Please mark these dates down and plan accordingly. Any conflicts must be legitimate and brought to my attention well before the exam is scheduled. If you have any specific learning disabilities or special needs and require accommodations, please let me know early in the semester so that your learning needs may be appropriately met. You will need to contact Dr. Neil Lipsitz in Disability Services (Hogan 209, x3693) to obtain documentation of your disability.

	Exam 1	**Thurs., Feb. 25**	**In Class**
Exam Schedule:	**Exam 2**	**Thurs., April 15**	**In Class**
	Final	**Sat., May 8**	**8:30 - 11:30 am**

Academic Integrity: The Department of Mathematics and Computer Science has drafted a policy on academic integrity to precisely state our expectations of both students and faculty with regards to cheating, plagiarism, academic honesty, etc. You are required to read this policy and sign a pledge agreeing to uphold it. Anyone who violates the Departmental Policy on Academic Integrity will receive a 0 for that assignment as well as possible further disciplinary action involving your Class Dean.

Grade: Your course grade will be determined as follows: homework 20%, computer projects 15%, final project 10%, midterm exams 30% and final exam 25%.

How to do well in this course:

- Attend class, participate and ask questions. Be an aggressive learner.
- Do your homework regularly.
- Read the text. (Yes, this is possible!)
- Work with your classmates.

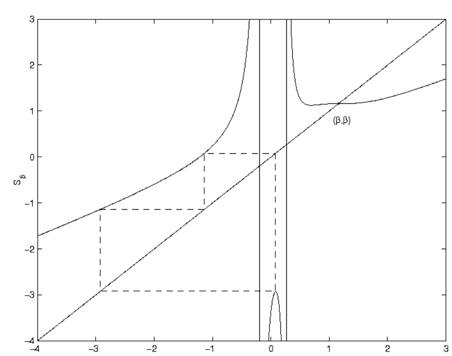

Figure 1: A super-attracting period 3 orbit.