Final Project

Dynamical Systems, Spring 2010

The last assignment of the course is to complete a final project focusing on some particular aspect or application related to the course material. Your project will consist of both a typed report (5 - 7 pages) and an in-class presentation (10 - 15 minutes) during the final week of class. Your report can be written using Maple (which has nice word processing for mathematical symbols) or a regular word processing program with a hand-written appendix for mathematical formulae. You will be allowed to work in groups of up to three people for the project (not necessarily the same group you've been working with on your Labs) although it is expected that each member will contribute equally. The final project is worth 10% of your total course grade.

Timeline and Due Dates:

- March 30: Brief description of final project topic, including names of group members, references and resources to be utilized
- April 13: Brief progress report detailing status of the project, including findings and further lines of inquiry
- April 23: Title of Final Project along with names of group members
- April 29, May 4: In-class Project Presentations (10 - 15 minutes)
- May 4: Final Report due (typed, 5 - 7 pages)

Some Topic Suggestions:

The aim of this project is for you to apply the mathematical knowledge gained from this course, as well as others, to explore a specific topic in dynamical systems or some application which uses the theory we have studied. This can involve reading research papers and presenting the results and/or doing actual mathematics in order to investigate a topic. For example, if a paper you are reading states that a period-doubling bifurcation occurs at a particular parameter value, then you should verify this. It is not expected that you will be doing ground-breaking research, but rather applying the tools you have developed in this course to a particular topic or application. Some sample topics are suggested below. Feel free to propose your own topic.

Caution: Be careful when using material found on the Internet. For example, some of the information on *Wikipedia* is correct and some is not. Be sure to check your findings thoroughly by confirming them with at least two independent, published (ie. peer-reviewed) sources. In general, your sources should be scholarly articles or books. It is suggested you utilize the powerful search engine MathSciNet (linked from the course webpage) to locate relevant materials.

1. **Newton's Method** There are equations that computers have a lot of trouble solving. For example, when applying Newton's method to certain polynomials, the iterative method may actually fail to find a root for an entire open set of initial guesses. Such bad polynomials are interesting to study from a dynamical systems perspective. Interesting fractals arise from applying Newton's method to complex polynomials.

Sample Resources: Chapter 13 of Devaney's text and "Newton's versus Halley's method: A dynamical systems approach," G. E. Roberts and J. Horgan-Kobelski, *International Journal of Bifurcation and Chaos*, Vol. 14, No. 10 (2004), 3459–3475.

2. **A Queueing Model** How do we model the decision-making process? Suppose that you have two tasks that must be completed in a certain amount of time. While you do one job, the other waits in the queue. There are some surprisingly complicated dynamics that can arise from such a simple system. Connections to the subject of economic dynamics are revealed in the nice paper by James Walsh, "Surprising Dynamics From a Simple Model," *Mathematics Magazine*, Vol. 79, No. 5 (2006), 327–339.

3. **Period Three Implies Chaos** Study the famous paper that ushered in the era of Chaos. What are the main theorems? How are they proven? Compare and contrast the proofs with those we did in class. "Period Three Implies Chaos," TY Li and J. A. Yorke, *American Mathematical Monthly*, Vol. 82, No. 10 (1975), 985–992.

4. **Applications to Population Dynamics** Where have biologists and scientists who try to model real-world populations needed dynamical systems theory? Find a particular model with parameters and study the bifurcations and dynamical behavior. What are the implications of the theory to the fate of the populations being studied?

   Sample Resources: "Biological Populations with Nonoverlapping Generations: Stable Points, Stable Cycles, and Chaos," Robert M. May, *Science*, Vol. 186, (1974), 645-647 and "Simple mathematical models with very complicated dynamics," Robert M. May, *Nature*, Vol. 261, June 10, 1976, 459–467.

5. **Applications to Medicine** How has the field of dynamical systems and chaos theory influenced medicine? Give specific examples, exploring the dynamical phenomena that occur.

   Sample Resources: "Fractal physiology and chaos in medicine," Bruce West, Studies of Nonlinear Phenomena in Life Science, 1, World Scientific Publishing Co., Inc., Teaneck, NJ, 1990, "Dynamical disease—the impact of nonlinear dynamics and chaos on cardiology and medicine," Leon Glass, The impact of chaos on science and society (Tokyo, 1991), 219–231, United Nations Univ. Press, Tokyo, 1997, and "Nearly One-Dimensional Dynamics in an Epidemic," W. M. Schaffer and M. Kot, *Journal of Theoretical Biology*, Vol. 112 (1985), 403–427.

6. **Applications to Economics** How has chaos theory influenced economics? Can we really use fractals to better understand the price of a given commodity or the ups and downs of the financial markets? Give specific examples including the relevant topics from dynamical systems being used.

   Sample Resources: "Chaos and nonlinear forecastability in economics and finance," B. LeBaron, *Philos. Trans. Roy. Soc. London Ser. A*, Vol. 348, No. 1688 (1994), 397–404 and "Chaos and chaotic dynamics in economics," M. Faggini, *Nonlinear Dyn. Psychol. Life Sci.*, Vol. 13, No. 3, (2009), 327–340.

7. **Fractal Geometry** How can fractals help deepen our understanding of the natural world? Where and how have the ideas of fractal geometry been successfully applied in other disciplines? What are the different ways to determine the fractal dimension of a set?

   Sample Resources: "The Fractal Geometry of Nature," Benoit Mandelbrot, W. H. Freeman and Company, New York, 1983, "A brief historical introduction to fractals and fractal geometry," Debnath, Lokenath, *Internat. J. Math. Ed. Sci. Tech.*, Vol. 37, No. 1 (2006), 29–50, and "Selected topics in mathematics, physics, and finance originating in fractal geometry," Benoit Mandelbrot, Thinking in Patterns, 1–33, World Sci. Publ., River Edge, NJ, 2004.

8. **Number Theory and Dynamical Systems** There are many interesting dynamical systems type problems in the field of number theory. HC's own Prof. Rafe Jones does research in this field, known as arithmetic dynamics. One nice example is the Ducci game given by the map

$$D(x_1, x_2, \ldots, x_n) \;=\; (|x_1 - x_2|, |x_2 - x_3|, \ldots, |x_n - x_1|)$$

   where $x_i \in \mathbb{Z}$ or $x_i \in \mathbb{Z}_m$. Study this map dynamically. What are the periodic points? What is the fate of most orbits? What are the open questions concerning this map?

   Sample Resources: "The N-number Ducci game," M. Chamberland, and D. Thomas, *J. Difference Equ. Appl.*, Vol. 10, No. 3 (2004), 339–342 and "A characterization for the length of cycles of the n-number Ducci game," N. Calkin, J. G. Stevens, D. Thomas, *Fibonacci Quart.*, Vol. 43, No. 1 (2005), 53–59.