Dynamical Systems, MATH 374

Homework Assignment #4

DUE DATE: Wed., Oct. 19th, Start of class.

You should write up solutions neatly to all problems, making sure to show all your work. A nonempty subset will be graded. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your own work.

Note: Please list the names of any students or faculty who you worked with on the assignment.

Problems on Topological Conjugacy

1. Suppose that f and g are topologically conjugate with conjugacy h, that is $h \circ f = g \circ h$. Show that h is a conjugacy between f^n and g^n for any $n \in \mathbb{N}$. In other words, show that

$$h \circ f^n = g^n \circ h \quad \forall n \in \mathbb{N}. \tag{1}$$

   Hint: Use the fact that h is invertible.

2. Use equation (1) to show that, if f and g are topologically conjugate, then h maps period n points of f to period n points of g. Specifically, show that p is a period n point for f if and only if $h(p)$ is a period n point for g.

3. Suppose that f and g are topologically conjugate and that p is an attracting fixed point for f. Prove that if x is asymptotically attracted to p under f, then $h(x)$ is asymptotically attracted to $h(p)$ under iteration of g. Conclude that h maps the basin of attraction of p to the basin of attraction of $h(p)$. (The *basin of attraction* of a fixed point is the set of all points attracted to the fixed point under iteration.)

4. Find a linear conjugacy $h(x) = \alpha x + \beta$ between the quadratic map $Q_c(x) = x^2 + c$ and the logistic map $F_\lambda(x) = \lambda x(1 - x)$. Give the values of α and β and the relationship between the parameters c and λ.

5. Using your conjugacy from the previous question, for what values of λ should we expect to see a period-doubling bifurcation occur for the logistic map.

6. Notice that $Q_{1/4}$ is conjugate to F_1 via the conjugacy from Problem 4. However, while Q_c undergoes a saddle-node bifurcation at $c = 1/4$, F_λ has a **different** bifurcation at $\lambda = 1$ (see Exercise 8, Ch. 6 from HW #3.) Why are the bifurcations for these two conjugate families different? By examining the relationship between the parameters c and λ given by the linear conjugacy, state precisely the values of the parameters for which Q_c is conjugate to F_λ.

Problems on Symbolic Dynamics

Chapter 9 Exercises (pp. 111 - 113)
Problems: 1, 2, 7, 8, 18a, 18b, 18f

Note: For the continuity problems in #18, the goal is to find a δ that works in the definition of continuity or convince yourself no such δ exists. For starters, go over the proof that the shift map is continuous from class.