MATH 305 Quiz #1 SOLUTIONS

February 11, 2016 **Prof. G. Roberts**

1. Let z be the complex number $\sqrt{3} - i$. (3 pts.)

   a) Write z in polar form, $z = re^{i\theta}$.
   Answer: $z = 2e^{-i\pi/6}$
   We have $r = \sqrt{(\sqrt{3})^2 + (-1)^2} = 2$ and using a 30-60-90 triangle, $\mathrm{Arg}(z) = -\pi/6$. (Note that z is in the fourth quadrant of the complex plane.)

   b) Using your answer to part **a)**, express z^9 in rectangular coordinates $x + iy$.
   Answer: $z^9 = 512i$.
   We have

$$z^9 \;=\; (2e^{-i\pi/6})^9 \;=\; 2^9 e^{-i9\pi/6} \;=\; 512e^{-i3\pi/2} \;=\; 512e^{i\pi/2} \;=\; 512(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}) \;=\; 512i.$$

2. Consider the following statements involving the conjugate and the modulus. Here, z, z_1, and z_2 are arbitrary complex numbers. Circle each statement that is always true. (4 pts.)

   a) $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$

   b) $|z_1 + z_2| = |z_1| + |z_2|$

   c) $z - \overline{z} = 2\,\mathrm{Re}(z)$

   d) $z \cdot \overline{z} = |z|^2$

   Answer: **a)** and **d)** are always true. **b)** should be $|z_1 + z_2| \leq |z_1| + |z_2|$ (the triangle *inequality*) while **c)** should be $z + \overline{z} = 2\,\mathrm{Re}(z)$.

3. Describe in words and sketch the set of all $z \in \mathbb{C}$ satisfying $|iz + 1| = 3$. (3 pts.)

   Answer: This is the circle centered at $z_0 = i$ of radius 3.

   The simplest way to do the problem is to factor out the i and use the property that $|z_1 z_2| = |z_1||z_2|$ (the modulus of the product equals the product of the moduli). We have

$$|iz + 1| \;=\; |i(z + \frac{1}{i})| \;=\; |i||z + \frac{1}{i}| \;=\; 1 \cdot |z - i|$$

   since $1/i = -i$. Thus, the equation reduces to $|z - i| = 3$. This is the set of all points that are 3 units from i, or a circle centered at $z_0 = i$ of radius 3.

   Note: Be sure to label the tickmarks on your axes when drawing curves and regions in the complex plane.