Complex Analysis

MATH 305, MWF 11:00 - 11:50, Swords 302, Fall 2011

Professor Gareth Roberts

Contacting me: Office: Haberlin 305, e-mail: `groberts@radius.holycross.edu` (Please use this email, it is **NOT** my groupwise account!) phone: x2350

Office hours: Mon. 1:00 - 2:00, Tues. 1:00 - 3:00, Wed. 8:00 - 8:50 (in Cool Beans) and 1:00 - 1:50, or by appointment.

Required Text: *Complex Variables and Applications*, James Ward Brown and Ruel V. Churchill

Course Prerequisite: MATH 242

Web page: `http://mathcs.holycross.edu/~groberts/Courses/MA305/homepage.html`
Homework assignments, projects, exam materials, schedule changes, useful links and other important information will be posted at this site. Please bookmark it!

Course objectives:

- Develop an understanding for the fundamental concepts in complex analysis.
- Demonstrate the ability to perform standard computations in complex analysis.
- Become proficient at making clear and coherent mathematical arguments.
- Work, communicate and share your knowledge with your peers.
- Have FUN learning complex analysis!

Syllabus: Complex analysis is a beautiful subject, with many surprising and profound results. This course will explore many of the concepts from one-variable calculus (limits, continuity, differentiation, integrals, power series) but applied to functions of a complex variable $z = x + i\,y$. Here, x and y are real numbers and i is the *imaginary* number that satisfies $i^2 = -1$. In many ways, studying the calculus of complex functions is much more natural and elegant than its real counterpart. For example, if a complex-valued function is differentiable on an open set in the complex plane, then it is actually infinitely-differentiable there. This is not the case for real-valued functions of a real variable.

Not only is the mathematics of complex analysis delightful, but there are many important applications to real-world problems, particularly involving ordinary and partial differential equations. One particular application we will discuss is *Laplace's equation*, which arises in certain physical cases such as heat flow and the motion of certain kinds of waves. Complex analysis is also a useful tool in other branches of mathematics such as algebra, number theory and dynamical systems. For example, one theorem we will study is *Liouville's Theorem* which can then be used to quickly prove the *Fundamental Theorem of Algebra*. Material pertaining to dynamical systems will be explored in the subsequent seminar titled *Complex Analytic Dynamics*.

Our study will be both computational and analytical. There will be many exercises reminiscent of calculus (e.g., find the value of a contour integral or compute an infinite series expansion). There will also be rigorous proofs demonstrated in class and assigned for homework. One of the strengths of complex analysis is the manner in which many deep facts in the subject can be easily verified.

We will cover much of the material in Chapters 1 - 7 of the course text by Brown and Churchill. We will also explore some applications as time permits. A rough outline of the semester is as follows:

- Complex Numbers: complex plane, complex conjugate, exponential form, argument, roots (4 classes)
- Analytic Functions: limits, continuity, derivatives, Cauchy-Riemann equations, harmonic functions (7 classes)
- Elementary Functions: exponential, logarithmic, branch cuts, trig functions, complex exponents (3 classes)
- Exam I (Chapters 1 - 3, some of 4)
- Integrals: definite integrals, contour integrals, Cauchy-Goursat theorem, Cauchy integral formula, Liouville's theorem, the fundamental theorem of algebra, maximum modulus principle (7 classes)
- Series: convergence, Taylor and Laurent series, uniform convergence, integration and differentiation of (5 classes)
- Residues and Poles: Cauchy's residue theorem, isolated singularities (removable, poles, essential), zeros (4 classes)
- Applications of Residues: real improper integrals, Fourier analysis, the argument principle (2 - 3 classes)
- Final Project Presentations (3 - 4 classes)
- Final Exam (Cumulative)

Homework: There will be regular homework assignments throughout the semester. Assignments will be posted on the course web page. There will also be occasional in-class worksheets or labs that will require the use of Maple. While you are allowed and encouraged to work on homework problems with your classmates, the solutions you turn in to be graded should be your own. Take care to write up solutions **in your own words**. Plagiarism will not be tolerated and will be treated as a violation of both the departmental policy on academic integrity and the college's policy on academic honesty.

NOTE: LATE homework will NOT be accepted. However, you will be allowed ONE "mulligan" over the course of the semester where you can turn in the assignment up to one week after the original due date.

Final Project: You are required to complete a final project focusing on some particular aspect or application of complex analysis. Examples of possible topics include minimal surfaces, fluid flow, temperature distributions and the Riemann zeta function. Details and suggestions of topics will be distributed later in the semester. Your project will include both a written report and an in-class presentation during the final week of class. You will work in small groups (2-3 people) for the project although it is expected that each member of the group will contribute significantly.

Exams and Quizzes: There will be one midterm exam on the evening of **Wednesday, October 26**, 7:00 - 8:30 pm, as well as a comprehensive final at the end of the semester. Any conflicts with the midterm exam or final must be legitimate and brought to my attention well before the exam is scheduled. In addition, there will be 3-4 short in-class quizzes throughout the semester designed to make sure that fundamental definitions, theorems and concepts are well-understood.

If you have any specific learning disabilities or special needs and require accommodations, please let me know early in the semester so that your learning needs may be appropriately met. You will need to contact the director of Disability Services (Hogan 215, x3693) to obtain documentation of your disability.

Academic Integrity: The Department of Mathematics and Computer Science has drafted a policy on academic integrity to precisely state our expectations of both students and faculty with regards to cheating, plagiarism, academic honesty, etc. You are required to read this policy and sign a pledge agreeing to uphold it. A violation of the Departmental Policy on Academic Integrity will result in a 0 for that assignment (or exam) and a letter describing the occurrence of academic dishonesty will be sent to your Class Dean.

Grade: Your course grade will be determined as follows: class participation/citizenship 5%, quizzes 5%, homework (including worksheets/labs) 20%, final project 20%, midterm exam 20% and final exam 30%.

How to do well in this course:

- Attend class, participate and ask questions. Be an engaged learner.
- Do your homework regularly.
- Read the text. (Yes, this is possible!)
- Work with your classmates.

Never regard study as a duty, but as the enviable opportunity to learn. – Albert Einstein

$$\text{Euler's formula:} \quad e^{i\theta} = \cos\theta + i\sin\theta$$

$$\text{Cauchy integral formula:} \quad f(z_0) = \frac{1}{2\pi i}\int_C \frac{f(z)}{z - z_0}\,dz$$

$$\text{Cauchy's residue theorem:} \quad \int_C f(z)\,dz = 2\pi i \sum_{k=1}^{n} \operatorname*{Res}_{z=z_k} f(z)$$