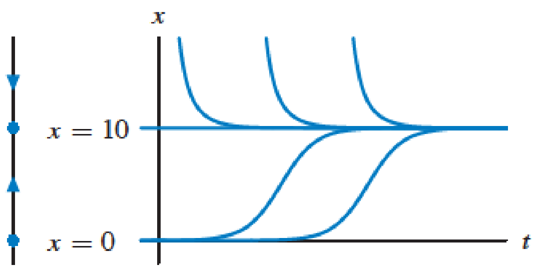
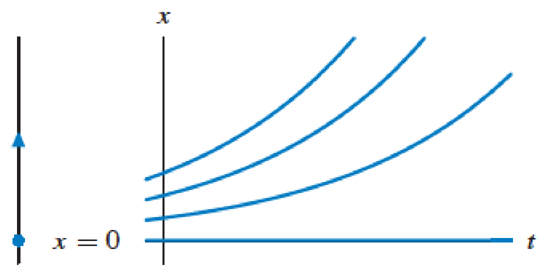
1. In the case where it takes many predators to eat one prey, the constant in the negative effect term of predators on the prey is small. Therefore, (ii) corresponds the system of large prey and small predators. On the other hand, one predator eats many prey for the system of large predators and small prey, and, therefore, the coefficient of negative effect term on predator-prey interaction on the prey is large. Hence, (i) corresponds to the system of small prey and large predators.

2. For (i), the equilibrium points are $x = y = 0$ and $x = 10, y = 0$. For the latter equilibrium point prey alone exist; there are no predators. For (ii), the equilibrium points are $(0, 0)$, $(0, 15)$, and $(3/5, 30)$. For the latter equilibrium point, both species coexist. For $(0, 15)$, the prey are extinct but the predators survive.

4. For (i), the prey obey a logistic model. The population tends to the equilibrium point at $x = 10$. For (ii), the prey obey an exponential growth model, so the population grows unchecked.

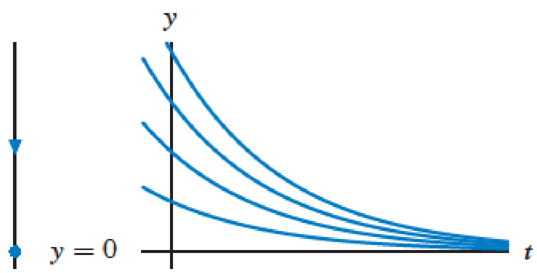


Phase line and graph for (i).

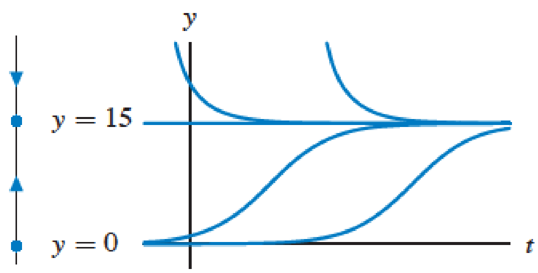


Phase line and graph for (ii).

6. For (i), the predators obey an exponential decay model, so the population tends to 0. For (ii), the predators obey a logistic model. The population tends to the equilibrium point at $y = 15$.

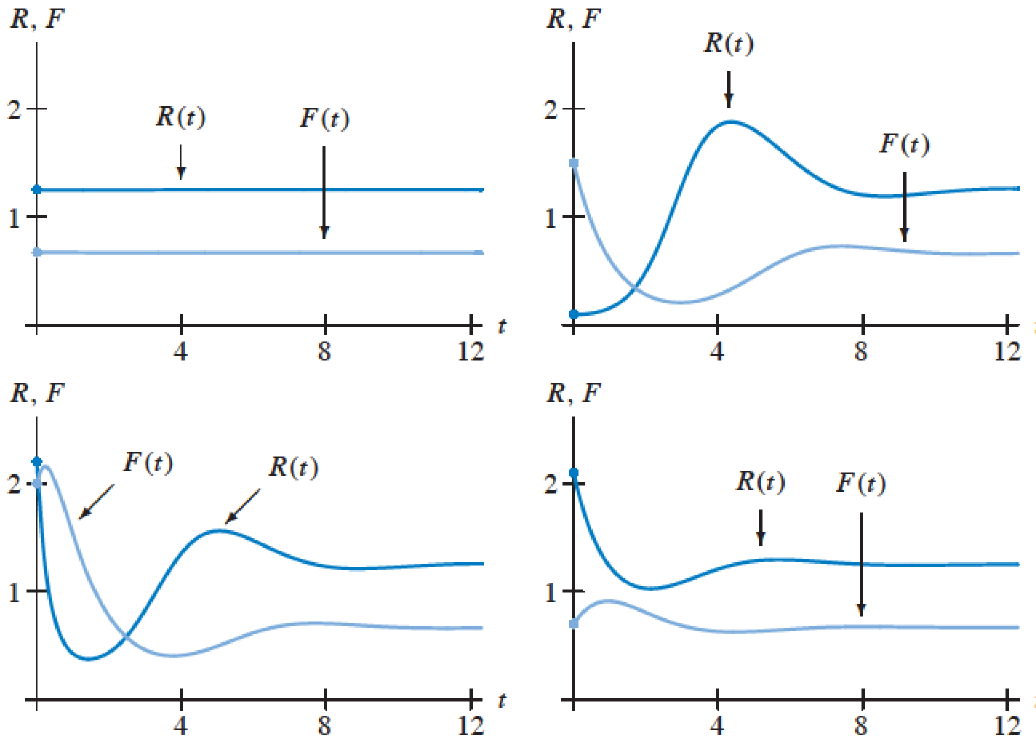


Phase line and graph for (i).



Phase line and graph for (ii).

8. **(a)**



(b) Each of the solutions tends to the equilibrium point at $(R, F) = (5/4, 2/3)$. The populations of both species tend to a limit and the species coexist. For curve B, note that the F-population initially decreases while R increases. Eventually F bottoms out and begins to rise. Then R peaks and begins to fall. Then both populations tend to the limit.

20. **(a)** If we substitute $y(t) = \cos \beta t$ into the left-hand side of the equation, we obtain

$$\begin{aligned} \frac{d^2y}{dt^2} + \frac{k}{m}y &= \frac{d^2(\cos \beta t)}{dt^2} + \frac{k}{m}\cos \beta t \\ &= -\beta^2 \cos \beta t + \frac{k}{m}\cos \beta t \\ &= \left(\frac{k}{m} - \beta^2\right)\cos \beta t \end{aligned}$$

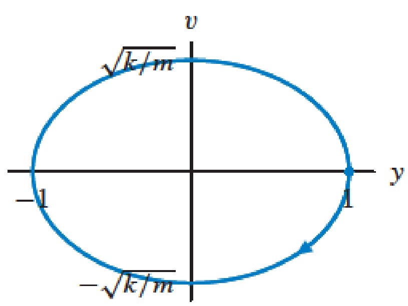
Hence, in order for $y(t) = \cos \beta t$ to be a solution we must have $k/m - \beta^2 = 0$. Thus,

$$\beta = \sqrt{\frac{k}{m}}.$$

(b) Substituting $t = 0$ into $y(t) = \cos \beta t$ and $v(t) = y'(t) = -\beta \sin \beta t$ we obtain the initial conditions $y(0) = 1$, $v(0) = 0$.

(c) The solution is $y(t) = \cos((\sqrt{k/m})t)$ and the period of this function is $2\pi/(\sqrt{k/m})$, which simplifies to $2\pi\sqrt{m}/\sqrt{k}$.

(d)



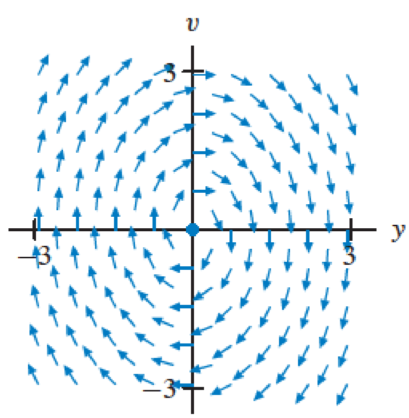
8. **(a)** Let $v = dy/dt$. Then

$$\frac{dv}{dt} = \frac{d^2y}{dt^2} = -2y.$$

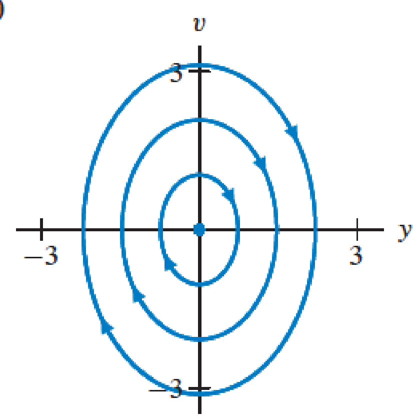
Thus the associated vector field is $\mathbf{V}(y, v) = (v, -2y)$.

(b) See part (c).

(c)

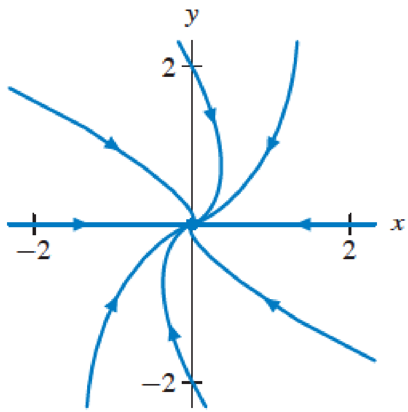


(d)



(e) As t increases, solutions move around the origin on ovals in the clockwise direction.

10. **(a)**



(b) The solution enters the first quadrant and tends to the origin tangent to the positive x-axis. Therefore $x(t)$ initially increases, reaches a maximum value, and then tends to zero as $t \to \infty$. It remains positive for all positive values of t. The function $y(t)$ decreases toward zero as $t \to \infty$.

11. **(a)** There are equilibrium points at $(\pm 1, 0)$, so only systems (ii) and (vii) are possible. Since the direction field points toward the x-axis if $y \neq 0$, the equation $dy/dt = y$ does not match this field. Therefore, system (vii) is the system that generated this direction field.

(b) The origin is the only equilibrium point, so the possible systems are (iii), (iv), (v), and (viii). The direction field is not tangent to the y-axis, so it does not match either system (iv) or (v). Vectors point toward the origin on the line $y = x$, so $dy/dt = dx/dt$ if $y = x$. This condition is not satisfied by system (iii). Consequently, this direction field corresponds to system (viii).

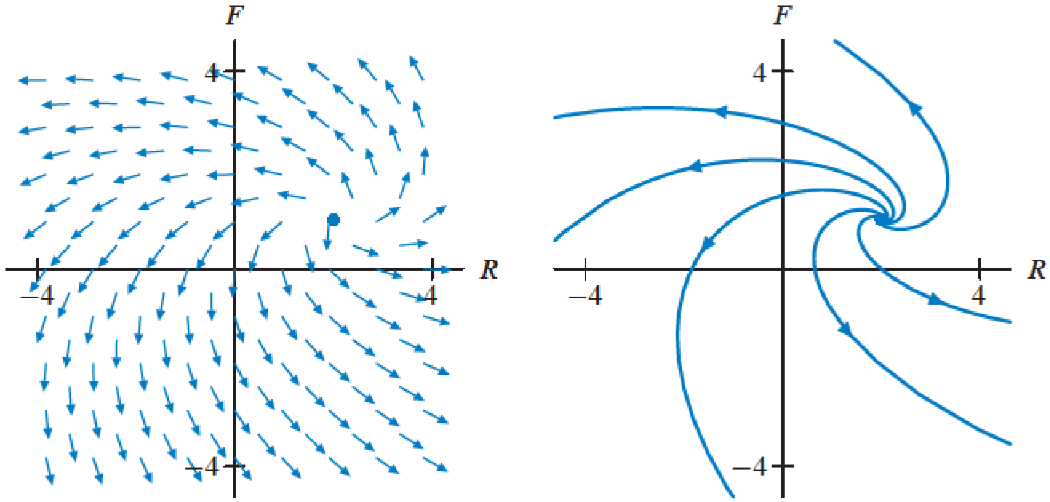
(c) The origin is the only equilibrium point, so the possible systems are (iii), (iv), (v), and (viii). Vectors point directly away from the origin on the y-axis, so this direction field does not correspond to systems (iii) and (viii). Along the line $y = x$, the vectors are more vertical than horizontal. Therefore, this direction field corresponds to system (v) rather than system (iv).

(d) The only equilibrium point is $(1, 0)$, so the direction field must correspond to system (vi).

14. **(a)** To find the equilibrium points, we solve the system of equations

$$\begin{cases} 4R - 7F - 1 = 0 \\ 3R + 6F - 12 = 0. \end{cases}$$

These simultaneous equations have one solution, $(R, F) = (2, 1)$.



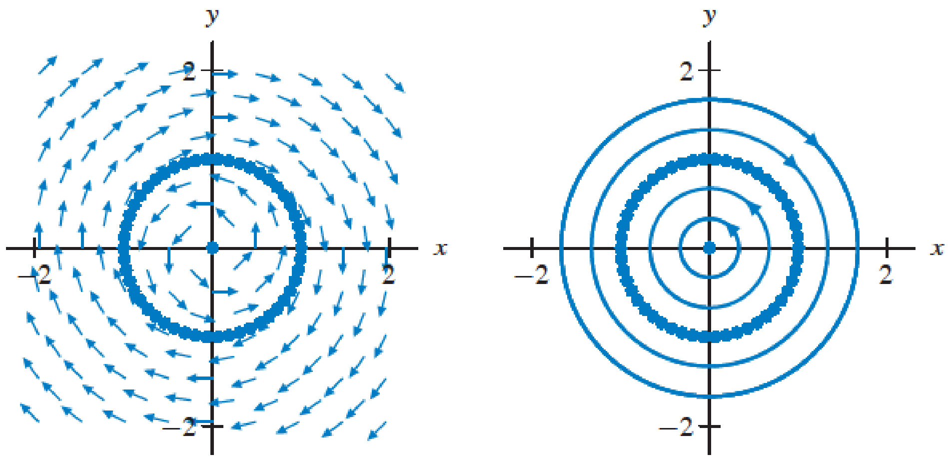
(b) As t increases, typical solutions spiral away from the equilibrium point at (2, 1)

18. **(a)** To find the equilibrium points, we solve the system of equations

$$\begin{cases} y(x^2+y^2-1)=0 \\ -x(x^2+y^2-1)=0. \end{cases}$$

If $x^2+y^2=1$, then both equations are satisfied. Hence, any point on the unit circle centered at the origin is an equilibrium point. If $x^2+y^2 \neq 1$, then the first equations implies $y=0$ and the second equation implies $x=0$. Hence, the origin is the only other equilibrium point.

(b)



(c) As t increases, typical solutions move on a circle around the origin, either counter-clockwise inside the unit circle, which consists entirely of equilibrium points, or clockwise outside the unit circle.

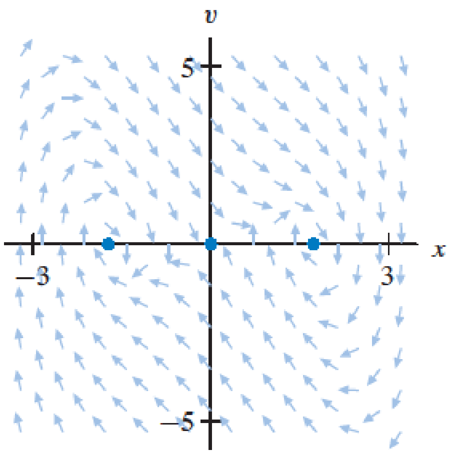
19. **(a)** Let $v = dx/dt$. Then

$$\frac{dv}{dt} = \frac{d^2x}{dt^2} = 3x - x^3 - 2v.$$

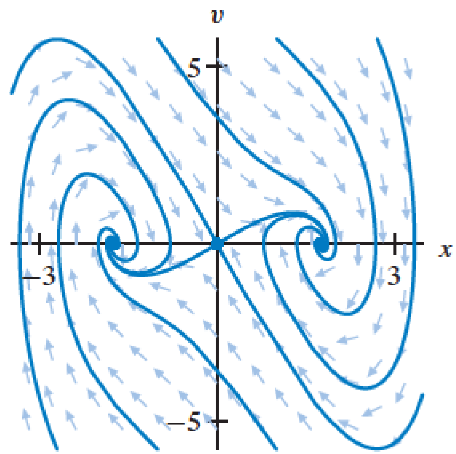
Thus the associated vector field is $\mathbf{V}(x, v) = (v, 3x - x^3 - 2v)$.

(b) Setting $\mathbf{V}(x, v) = (0, 0)$ and solving for (x, v), we get $v = 0$ and $3x - x^3 = 0$. Hence, the equilibria are $(x, v) = (0, 0)$ and $(x, v) = (\pm\sqrt{3}, 0)$.

(c)

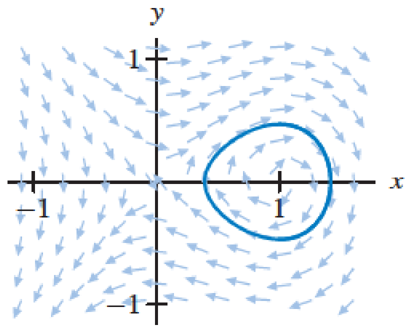


(d)

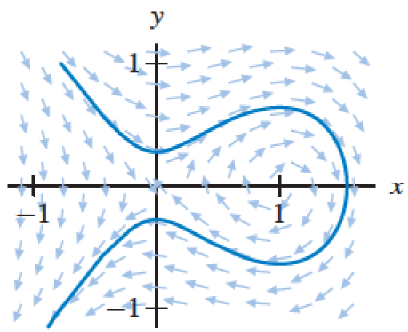


(e) As t increases, almost all solutions spiral to one of the two equilibria $(\pm\sqrt{3}, 0)$. There is a curve of initial conditions that divides these two phenomena. It consists of those initial conditions for which the corresponding solutions tend to the equilibrium point at $(0, 0)$.

21. **(a)** The $x(t)$- and $y(t)$-graphs are periodic, so they correspond to a solution curve that returns to its initial condition in the phase plane. In other words, its solution curve is a closed curve. Since the amplitude of the oscillation of $x(t)$ is relatively large, these graphs must correspond to the outermost closed solution curve.

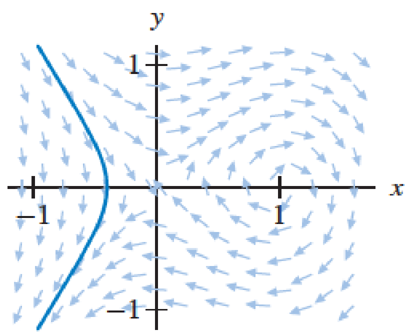


(b) The graphs are not periodic, so they cannot correspond to the two closed solution curves in the phase portrait. Both graphs cross the t-axis. The value of $x(t)$ is initially negative, then becomes positive and reaches a maximum, and finally becomes negative again. Therefore, the corresponding solution curve is the one that starts in the second quadrant, then travels through the first and fourth quadrants, and finally enters the third quadrant.

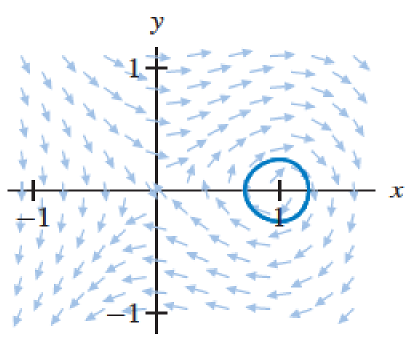


(c) The graphs are not periodic, so they cannot correspond to the two closed solution curves in the phase portrait. Only one graph crosses the t-axis. The other graph remains negative for all time. Note that the two graphs cross.

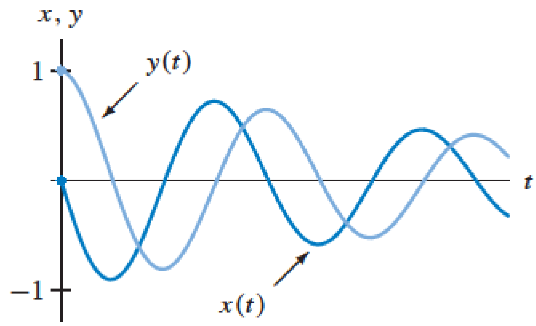
The corresponding solution curve is the one that starts in the second quadrant and crosses the x-axis and the line $y = x$ as it moves through the third quadrant.



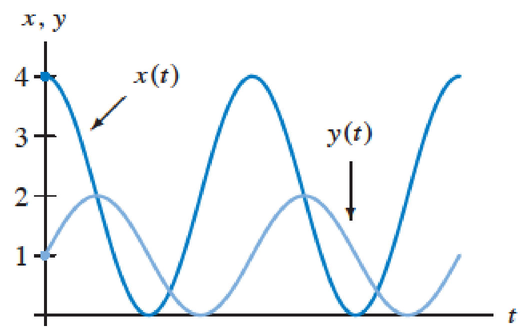
(d) The $x(t)$- and $y(t)$-graphs are periodic, so they correspond to a solution curve that returns to its initial condition in the phase plane. In other words, its solution curve is a closed curve. Since the amplitude of the oscillation of $x(t)$ is relatively small, these graphs must correspond to the innermost closed solution curve.



23. Since the solution curve spirals into the origin, the corresponding $x(t)$- and $y(t)$-graphs must oscillate about the t-axis with the decreasing amplitudes.



24. Since the solution curve is an ellipse that is centered at $(2, 1)$, the $x(t)$- and $y(t)$-graphs are periodic. They oscillate about the lines $x = 2$ and $y = 1$.



2. **(a)** See part (c).

(b) We guess that there are solutions of the form $y(t) = e^{st}$ for some choice of the constant s. To determine these values of s, we substitute $y(t) = e^{st}$ into the left-hand side of the differential equation, obtaining

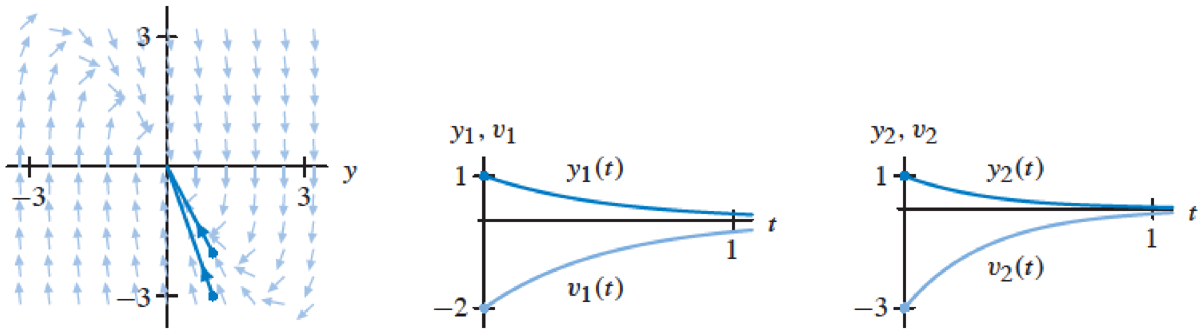
$$\begin{aligned}\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y &= \frac{d^2(e^{st})}{dt^2} + 5\frac{d(e^{st})}{dt} + 6(e^{st}) \\ &= s^2e^{st} + 5se^{st} + 6e^{st} \\ &= (s^2 + 5s + 6)e^{st}\end{aligned}$$

In order for $y(t) = e^{st}$ to be a solution, this expression must be 0 for all t. In other words,

$$s^2 + 5s + 6 = 0.$$

This equation is satisfied only if $s = -3$ or $s = -2$. We obtain two solutions, $y_1(t) = e^{-3t}$ and $y_2(t) = e^{-2t}$, of this equation.

(c)



3. **(a)** See part (c).

(b) We guess that there are solutions of the form $y(t) = e^{st}$ for some choice of the constant s. To determine these values of s, we substitute $y(t) = e^{st}$ into the left-hand side of the differential equation, obtaining

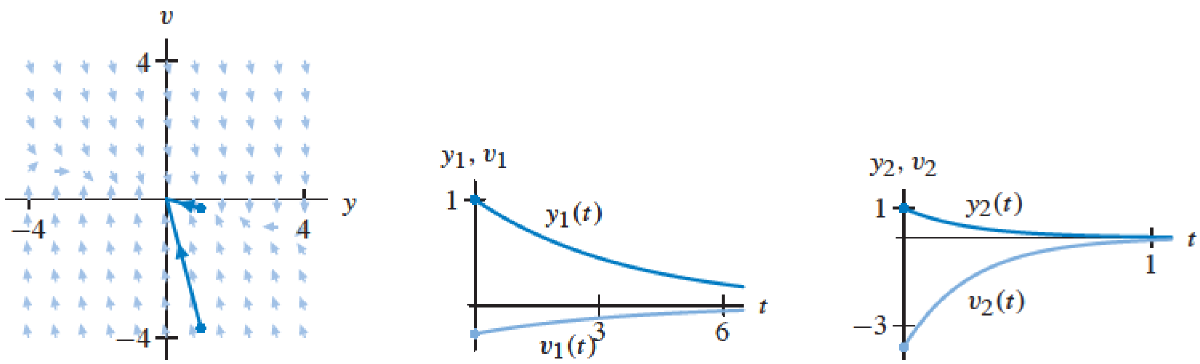
$$\begin{aligned}\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + y &= \frac{d^2(e^{st})}{dt^2} + 4\frac{d(e^{st})}{dt} + e^{st} \\ &= s^2e^{st} + 4se^{st} + e^{st} \\ &= (s^2 + 4s + 1)e^{st}\end{aligned}$$

In order for $y(t) = e^{st}$ to be a solution, this expression must be 0 for all t. In other words,

$$s^2 + 4s + 1 = 0.$$

Applying the quadratic formula, we obtain the roots $s = -2 \pm \sqrt{3}$ and the two solutions, $y_1(t) = e^{(-2-\sqrt{3})t}$ and $y_2(t) = e^{(-2+\sqrt{3})t}$, of this equation.

(c)



5. **(a)** See part (c).

(b) We guess that there are solutions of the form $y(t) = e^{st}$ for some choice of the constant s. To determine these values of s, we substitute $y(t) = e^{st}$ into the left-hand side of the differential equation, obtaining

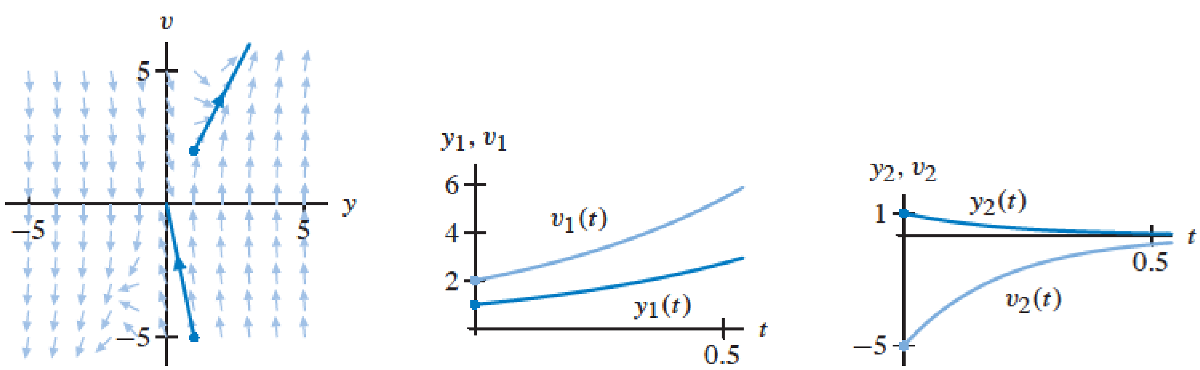
$$\begin{aligned}\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - 10y &= \frac{d^2(e^{st})}{dt^2} + 3\frac{d(e^{st})}{dt} - 10(e^{st}) \\ &= s^2e^{st} + 3se^{st} - 10e^{st} \\ &= (s^2 + 3s - 10)e^{st}\end{aligned}$$

In order for $y(t) = e^{st}$ to be a solution, this expression must be 0 for all t. In other words,

$$s^2 + 3s - 10 = 0.$$

This equation is satisfied only if $s = -5$ or $s = 2$. We obtain two solutions, $y_1(t) = e^{-5t}$ and $y_2(t) = e^{2t}$, of this equation.

(c)



3. To check that $dx/dt = 2x + 2y$, we compute both

$$\frac{dx}{dt} = 2e^t - 4e^{4t}$$

and

$$2x + 2y = 4e^t - 2e^{4t} - 2e^t + 2e^{4t} = 2e^t.$$

Since the results of these two calculations do not agree, the first equation in the system is not satisfied, and $(x(t), y(t))$ is not a solution.

4. To check that $dx/dt = 2x + 2y$, we compute both

$$\frac{dx}{dt} = 4e^t + 4e^{4t}$$

and

$$2x + 2y = 8e^t + 2e^{4t} - 4e^t + 2e^{4t} = 4e^t + 4e^t.$$

To check that $dy/dt = x + 3y$, we compute both

$$\frac{dy}{dt} = -2e^t + 4e^{4t},$$

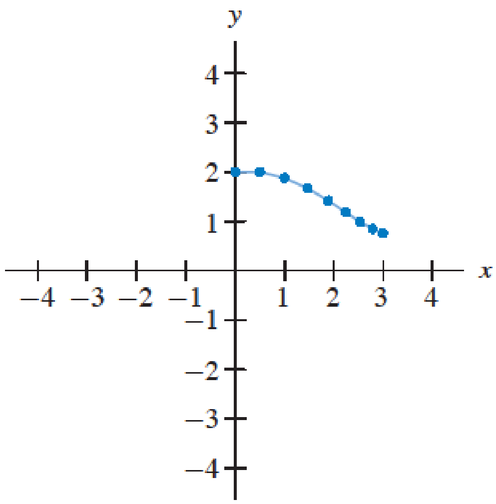
and

$$x + 3y = 4e^t + e^{4t} - 6e^t + 3e^{4t} = -2e^t + 4e^{4t}.$$

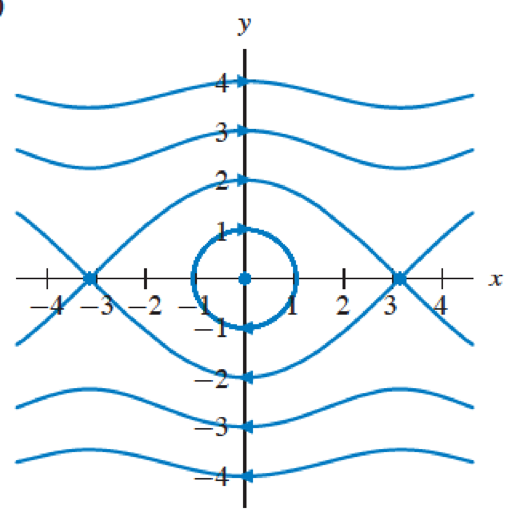
Both equations are satisfied for all t. Hence $(x(t), y(t))$ is a solution.

4. **(a)** Euler approximation yields $(x_8, y_8) \approx (3.00, 0.76)$.

(b)

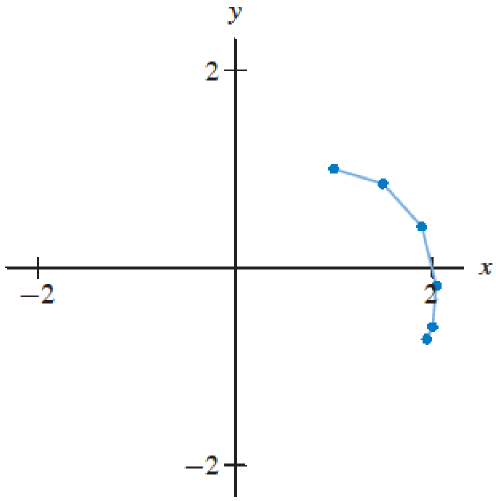


(c)



5. **(a)** Euler approximation yields $(x_5, y_5) \approx (1.94, -0.72)$.

(b)



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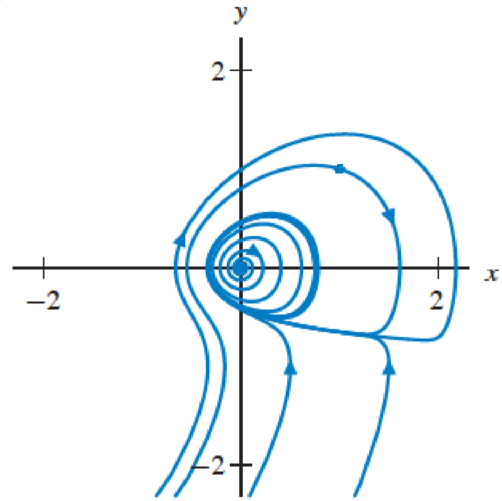
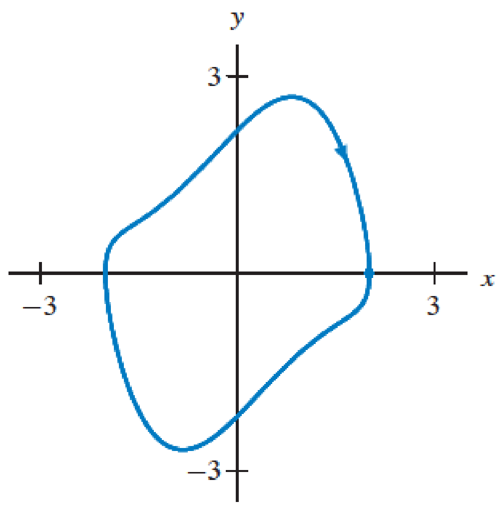


2. **(a)** There are infinitely many initial conditions that yield a periodic solution. For example, the initial condition $(2.00, 0.00)$ lies on a periodic solution.



(b) Any solution with an initial condiion that is inside the periodic curve is trapped for all time. Namely, the period solution forms a "fence" that stops any solution with an initial condition that is inside the closed curve from "escaping." Since the system is autonomous, no nonperiodic solution can touch the solution curve for this period solution.

8. **(a)** Differentiation yields

$$\frac{d\mathbf{Y}_2}{dt} = \frac{d(\mathbf{Y}_1(t+t_0))}{dt} = \mathbf{F}(\mathbf{Y}_1(t+t_0)) = \mathbf{F}(\mathbf{Y}_2(t))$$

where the second equality uses the Chain Rule and the other two equalities involve the definition of $\mathbf{Y}_2(t)$.

(b) They describe the same curve, but differ by a constant shift in parameterization.