Ordinary Differential Equations

MATH 304, MWF 12:00 - 12:50, Swords 302, Fall 2014

Professor Gareth Roberts

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Office Hours: Mon. 2:00 - 3:00, Wed. 2:00 - 3:00, Thurs. 1:00 - 2:30, Fri. 1:00 - 2:00, or by appointment.

Required Text: *Differential Equations*, Fourth ed., Paul Blanchard, Robert Devaney and Glen Hall

Course Prerequisites: MATH 242 (Principles of Analysis) and MATH 244 (Linear Algebra)

Webpage: `http://mathcs.holycross.edu/~groberts/Courses/MA304/homepage.html`
Homework assignments, computer projects, schedule changes, exam materials, useful links and other important information will be posted at this site. Please bookmark it!

Course Objectives:

- Learn to apply and synthesize the analytic, qualitative and numerical techniques for solving differential equations.
- Become proficient at making clear and coherent mathematical arguments.
- Work and communicate with your peers.
- Have FUN learning differential equations!

Course Content: This is an applied mathematics course focusing on ordinary differential equations, that is, equations relating the derivatives of unknown functions to themselves. For example, the differential equation $dy/dt = 2y^2 - y$ relates the derivative of the unknown function $y(t)$ to a quadratic expression of the function itself $2y(t)^2 - y(t)$. Another example, well-known to engineers, is the equation for a damped harmonic oscillator with external forcing:

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky \;=\; a\cos(\omega t).$$

Here, the unknown function $y(t)$ represents the position of a mass m at time t attached to a mounted spring. There are literally thousands of differential equations used to model phenomena in all kinds of fields including physics (mechanics), chemistry (reactions), biology (population models), neuroscience (brain functioning), economics (the Black-Scholes equation) astronomy (space craft trajectories), and fluid dynamics (turbulence). Any natural process that involves a change over time can be modeled with a differential equation.

We will utilize three different approaches to studying ordinary differential equations (ODEs): analytic, numerical and qualitative techniques. Since most differential equations are too large in dimension or too difficult to solve explicitly, qualitative methods play an important role in modern research. With the rapid growth in technology, using computers to approximate and visualize solutions has also become increasingly common. In order for you to obtain an appreciation for this aspect of the field, several computer projects using the software package DE Tools (included with the course text) will be assigned.

We will cover most of the material from Chapters 1 through 5 and then finish with a brief tour of one of my favorite subjects, celestial mechanics. Other topics such as Picard iteration, the Poincaré map and dynamical systems theory will be covered if time permits. A rough outline of the semester follows:

- Introduction to ODEs and overview of the course (1 class)
- First-Order Equations: population models, separation of variables, slope fields, Euler's method, existence and uniqueness theorems, phase lines, bifurcations, solving linear equations (11 classes)
- Exam I
- Systems: physical examples, vector fields, Euler's method, the Lorenz equations (5 classes)
- Planar Linear Systems: eigenvalues and eigenvectors, analytic solutions, sketching phase planes, the trace-determinant plane (8 classes)
- The Harmonic Oscillator: 2nd-order equations, forced oscillators, resonance (3 classes)
- Exam II
- Nonlinear Systems: equilibria, stability, nullclines, Hamiltonian systems (4 classes)
- Celestial Mechanics: the Kepler problem, the n-body problem, special solutions (3 classes)
- Final Exam

Homework: There will be homework due every Friday at the START of class, except for weeks when a midterm exam takes place. Assignments will be posted on the course web page. There will be a list of problems assigned, a nonempty subset of which will be graded. Solutions to all problems will be posted immediately after the assignment is due.

While you are allowed and encouraged to work on homework problems with your classmates, the solutions you turn in to be graded should be your own. Take care to write up solutions **in your own words**. Plagiarism will not be tolerated and will be treated as a violation of both the departmental policy on academic integrity and the college's policy on academic honesty.

NOTE: LATE homework will NOT be accepted.

Computer Projects: There will be several computer projects assigned over the course of the semester using the software package DE Tools included with the course text. Each project will explore some real world application of differential equations and may require some supplementary reading. Projects will be completed in groups of 2 to 3 people with one report to be turned in for the entire group.

Exams: There will be two evening midterms and a comprehensive final at the end of the semester. Please make a note of these dates and plan accordingly. Any conflicts must be legitimate and brought to my attention well before the scheduled exam date. If you have any specific learning disabilities or special needs and require accommodations, please let me know early in the semester so that your learning needs may be appropriately met. You will need to contact the director of Disability Services in Hogan 209 (x 3693) to obtain documentation of your disability. We will review for the midterms during the Monday class on the week of the exam.

	Exam 1	**Wed., Oct. 8**	**evening**
Exam Schedule:	**Exam 2**	**Wed., Nov. 19**	**evening**
	Final	**Thurs., Dec. 18**	**11:30 am - 2:00 pm**

Academic Integrity: The Department of Mathematics and Computer Science has drafted a policy on academic integrity to precisely state our expectations of both students and faculty with regards to cheating, plagiarism, academic honesty, etc. You are required to read this policy and sign a pledge agreeing to uphold it. A violation of the Departmental Policy on Academic Integrity will result in a 0 for that assignment (or exam) and a letter describing the occurrence of academic dishonesty will be sent to your Class Dean.

Grade: Your course grade will be based on the following breakdown:

- classroom participation/interest 5%
- homework and computer projects 35%
- midterm exams 35%
- final exam 25%

How to do well in this course:

- Attend class, participate and ask questions.
- Work with your classmates. Organize study groups.
- Be an active, engaged learner.
- Do your homework regularly.
- Read the text. (It even has a few jokes!)

Never regard study as a duty, but as the enviable opportunity to learn.

Albert Einstein