

Working with Data: Soil Respiration at Harvard Forest

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Figure: Soil Warming Experiment at **Harvard Forest**. Cables are buried in different plots to heat (or not) the soil 5°C above the ambient temperature in order to determine the effect on soil CO_2 emissions. Figure Source: <http://harvardforest.fas.harvard.edu/photos/soil-warming-control-plot>

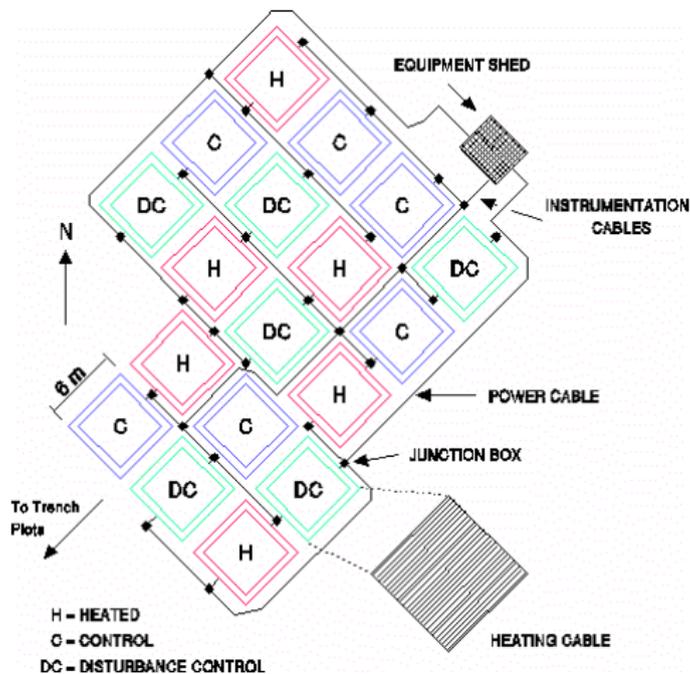


Figure: Experiment begun in 1991 consisting of 18 total plots, each $6 \times 6 \text{ m}^2$, of three different types: H (heated via buried cables), C (control, undisturbed with no cables), DC (disturbed control, buried cables with no heat). DC plots used to distinguish effects from heating versus cable installation.

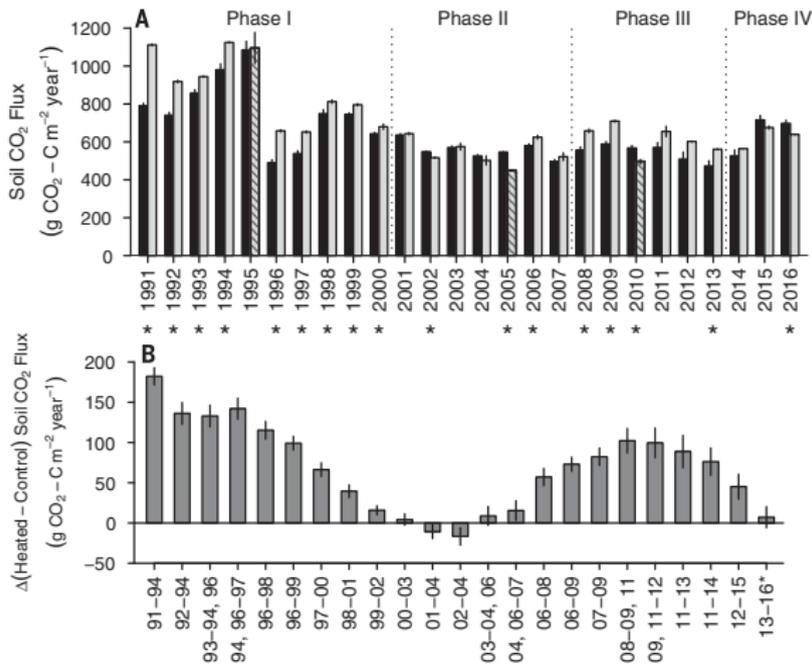


Figure: Figure 1 from “Long-term pattern and magnitude of soil carbon feedback to the climate system in a warming world,” Melillo et. al., *Science* **358**, 101–105 (2017). (A) shows soil CO₂ emissions from control plots (black) compared with heated plots (gray). (B) gives the four-year rolling mean change in emissions (heated – control).

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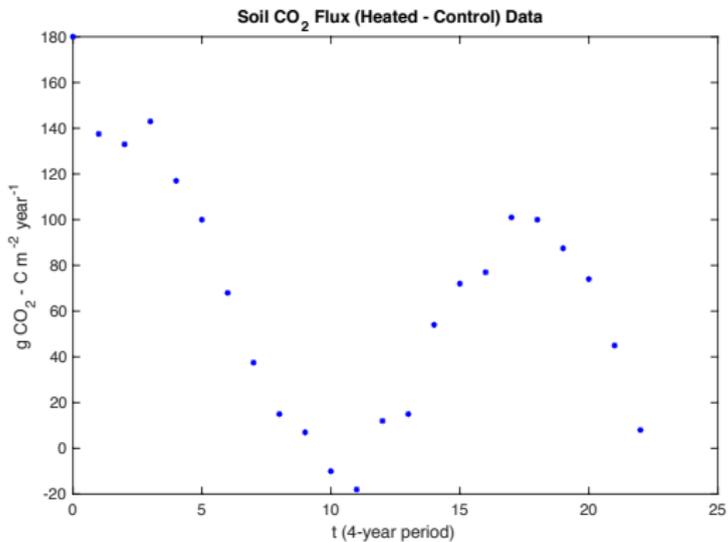
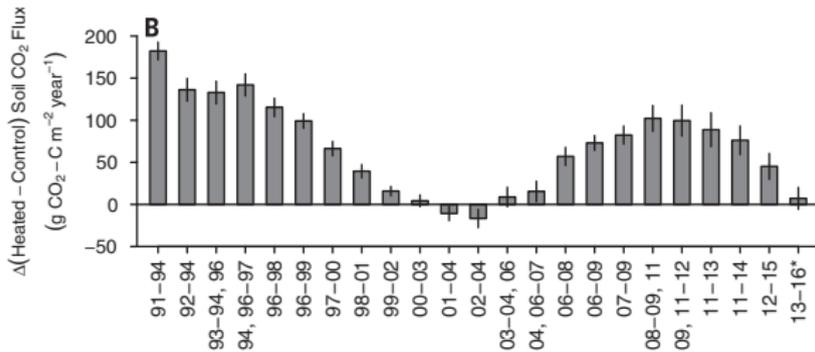
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- 5 Over the full 26-year study, 17% of the soil carbon contained in the top 60 cm of the heated soil was lost.



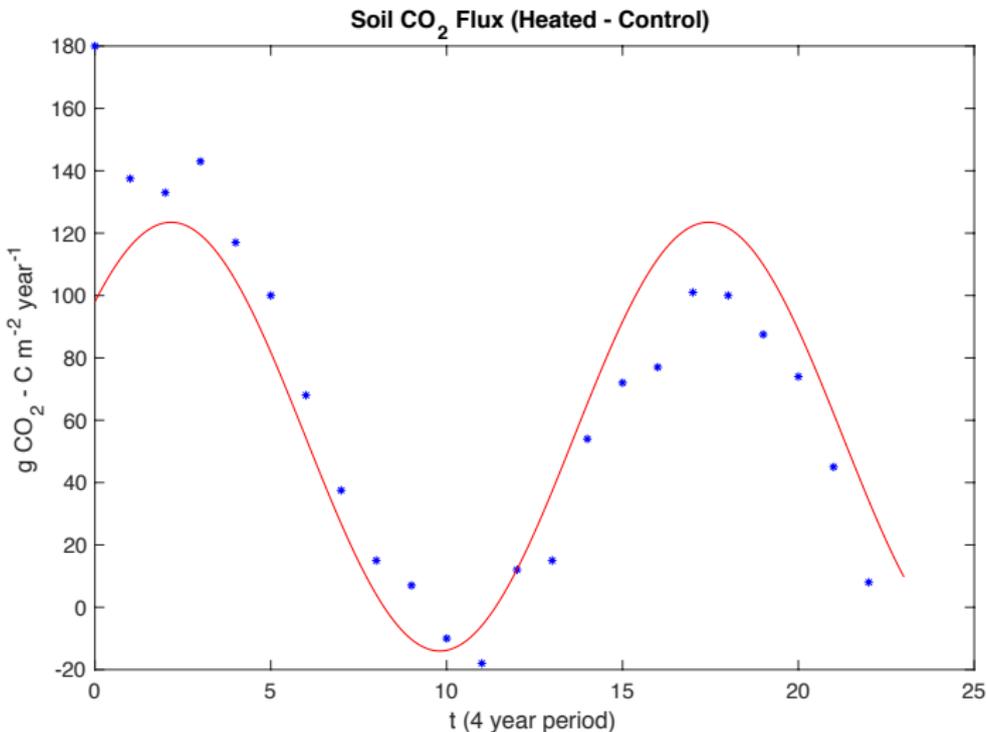


Figure: Cosine fit to soil CO₂ flux data. $F = a\cos(b(t - c)) + d$, where $a = 68.76$, $b = 0.41$, $c = 2.16$, and $d = 54.72$. The R^2 -value is 0.7821 and the period of the fit is 15.28 years.

Matlab Code

```
format long;
plot(t,F,'b*'); %% Data Plot: t = years, F = CO2 flux values

per = 15.28; %% Period; estimated to minimize R^2 value
b = 2*pi/per;

%% Least squares approach to fit standard cosine curve
X = [cos(b*t) sin(b*t) ones(23,1)]; %% Least-squares matrix
v = inv(X'*X)*X'*F; %% Least-squares solution

a = sqrt(v(1)^2 + v(2)^2); %% amplitude of cosine fit
c = atan(v(2)/v(1))/b; %% phase shift
d = v(3); %% vertical shift

s = [0:0.1:23];
y = a*cos(b*(s-c)) + d;
plot(t,F,'b*',s,y,'r'); %% Plot of data and fit together

FHat = X*v;
Rsqr = (norm(FHat-mean(F))/norm(F-mean(F)))^2;
%% R^2 value is 0.7821
```