Budyko's Energy Balance Model

Gareth E. Roberts

Department of Mathematics and Computer Science
College of the Holy Cross
Worcester, MA, USA

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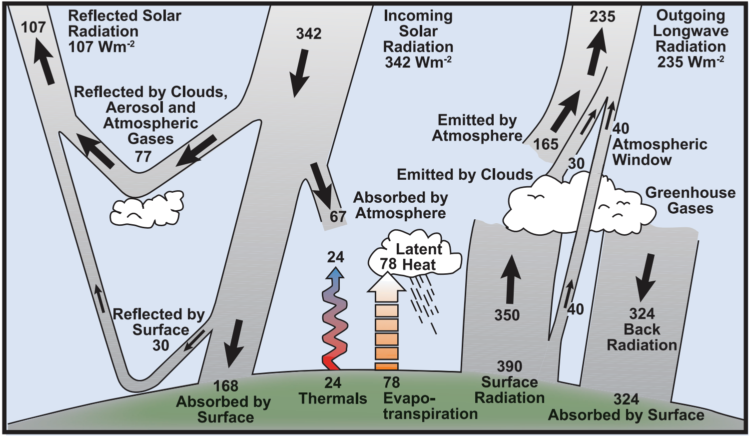

FAQ 1.1, Figure 1. *Estimate of the Earth's annual and global mean energy balance. Over the long term, the amount of incoming solar radiation absorbed by the Earth and atmosphere is balanced by the Earth and atmosphere releasing the same amount of outgoing longwave radiation. About half of the incoming solar radiation is absorbed by the Earth's surface. This energy is transferred to the atmosphere by warming the air in contact with the surface (thermals), by evapotranspiration and by longwave radiation that is absorbed by clouds and greenhouse gases. The atmosphere in turn radiates longwave energy back to Earth as well as out to space. Source: Kiehl and Trenberth (1997).*

Figure: Heat Balance. Recall: $Q = S/4 = 342 \text{ W/m}^2$. Source: "Historical Overview of Climate Change Science," IPCC AR4, (2007) p. 96.

Tilt of the Earth

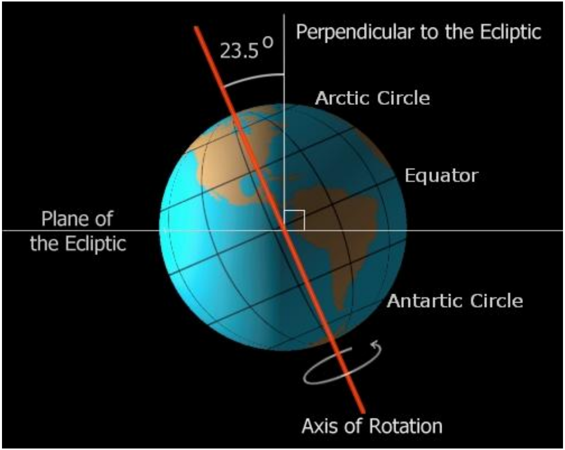

Figure: The Earth is tilted (obliquity) 23.5° from the normal to the plane of the ecliptic (the plane the planets travel in around the sun). The obliquity changes on a 40,000 year cycle. Source: `http://www.rsd17.org/TeacherWebPage/HighSchool/JAnderson/A/introduction/earthinspace/earthsTilt.jpg`

Insolation Distribution

green = quadratic approximation (Chylek & Coakley)

fuchsia = formula using obliquity of 23.5°

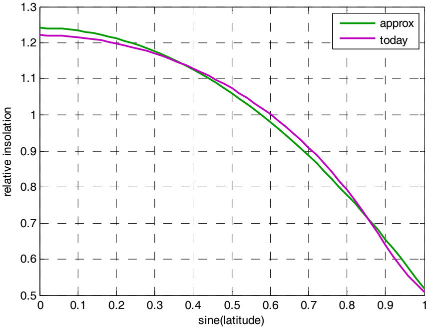

Figure: The quadratic approximation to the insolation distribution $s(y)$ is quite good.

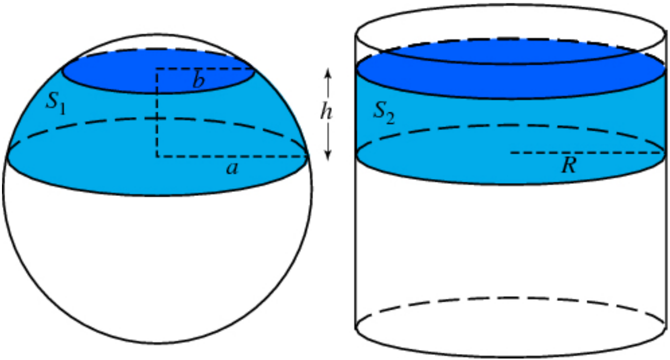

Figure: Archimedes' Hat-Box Theorem: $S_1 = S_2 = 2\pi Rh$. The cylinder and sphere have the same radius ($a = R$). Think of the sphere being circumscribed by the cylinder.

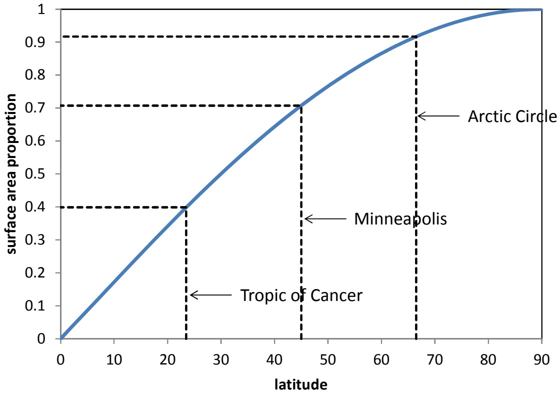

Figure: A plot of $y = \sin\theta$ along with some key latitudes. Due to Archimedes' Hat-Box Theorem, the proportion of the Earth's surface area from the equator to a given latitude θ is simply $y/2$, and between $-\theta$ and θ it is just y.

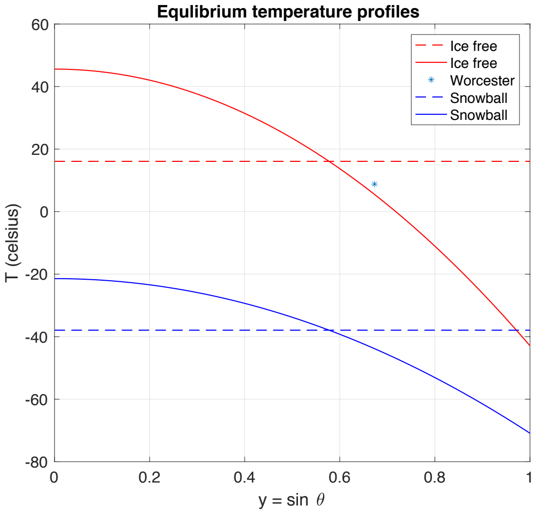

Figure: Graphs of equilibrium temperatures with (solid) and without (dashed) latitude dependence. Ice free is $\alpha = 0.32$ (red); snowball Earth is $\alpha = 0.62$ (blue). Note that incorporating latitude allows ice caps to form in the ice free case.

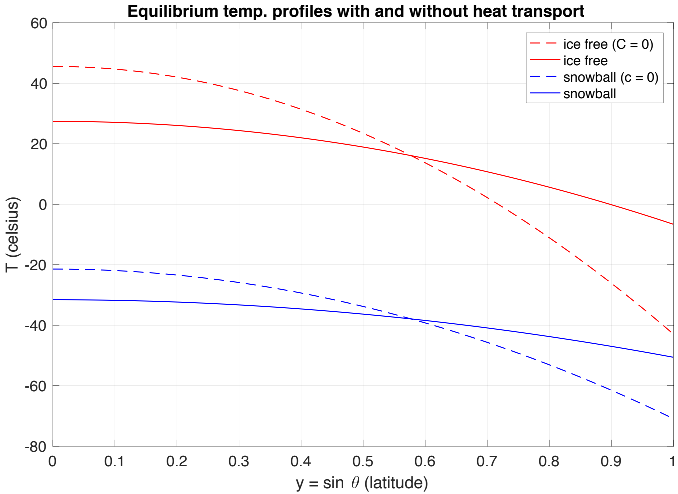

Figure: Graphs of equilibrium temperatures with ($C = 3.04$; solid) and without ($C = 0$; dashed) heat transport. Ice free is $\alpha = 0.32$ (red); snowball Earth is $\alpha = 0.62$ (blue).

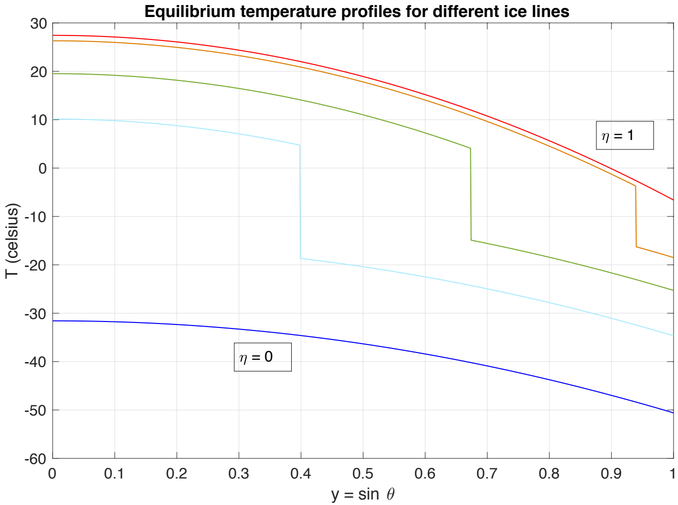

Figure: Graphs of equilibrium temperatures with two-step albedo function for different ice lines: $\eta = 1$ (red; ice free), $\eta = \sin(70^\circ)$ (orange; current), $\eta = \sin(42.3^\circ)$ (green; Worcester), $\eta = \sin(23.5^\circ)$ (light blue; Tropic of Cancer), $\eta = 0$ (blue; snowball).

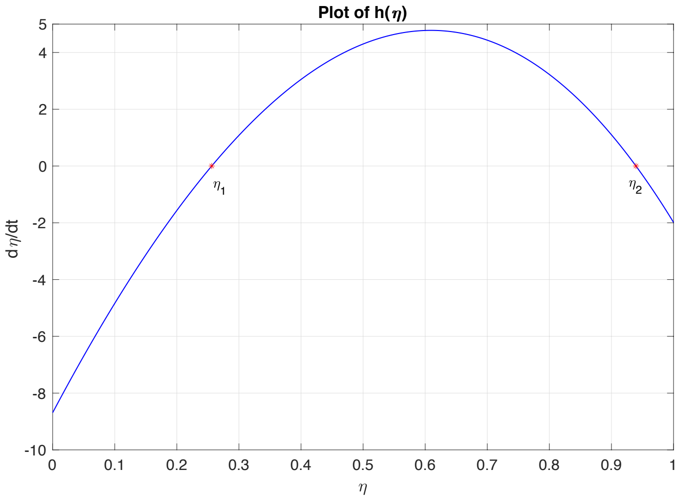

Figure: Plot of $h(\eta)$ for the Widiasih ice-line equation $d\eta/dt = \epsilon h(\eta)$ showing two equilibria ice line positions at $\eta_1 \approx 0.2562$ (unstable) and $\eta_2 \approx 0.9394$ (stable).

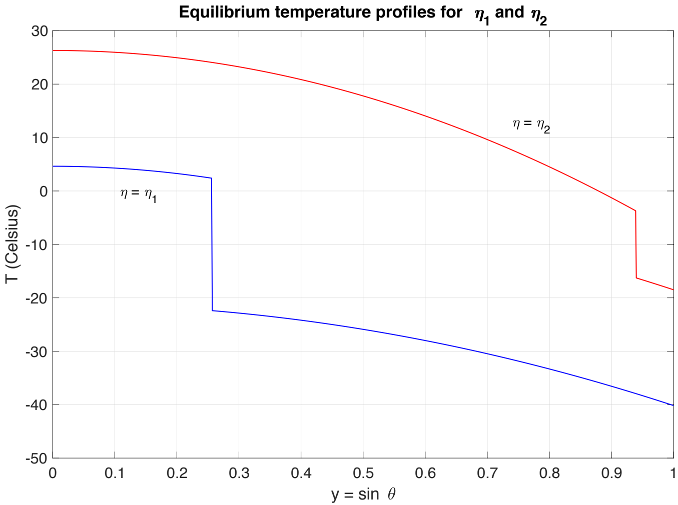

Figure: Equilibrium temperature profiles for the two ice line equilibrium points $\eta = \eta_1 \approx 0.2562$ (blue) and $\eta = \eta_2 \approx 0.9394$ (red). The red curve is very close to our current climate.

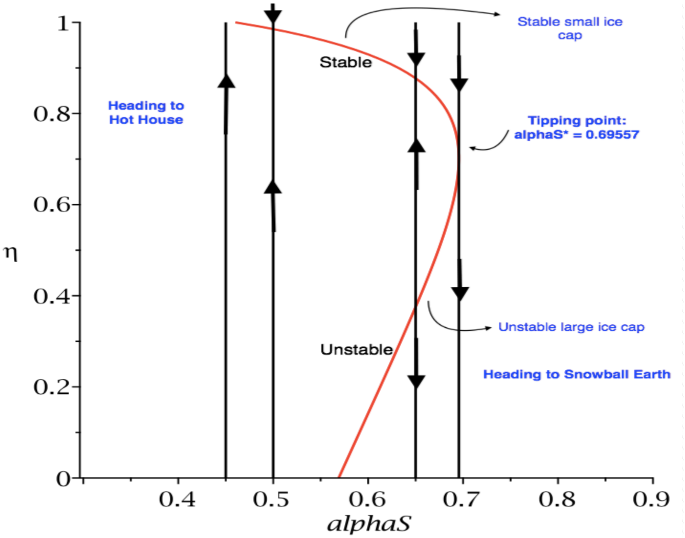

Figure: Bifurcation diagram showing the location of the ice line equilibria (roots of $h(\eta)$) as the albedo parameter α_s is varied. Note the tipping point at $\alpha_s \approx 0.69557$. Figure by Cara Donovan.

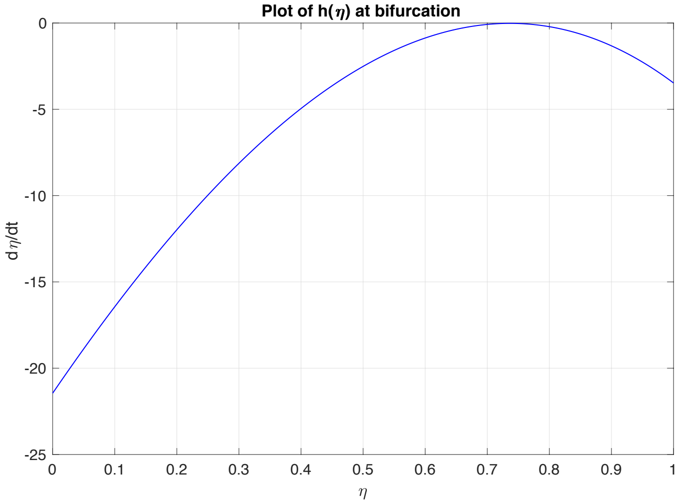

Figure: Plot of $h(\eta)$ for $\alpha_s = \alpha_s^*$ showing saddle-node (tangent) bifurcation. Once α_s increases above α_s^*, there are no equilibria and the ice line decreases toward the equator (Snowball Earth scenario).

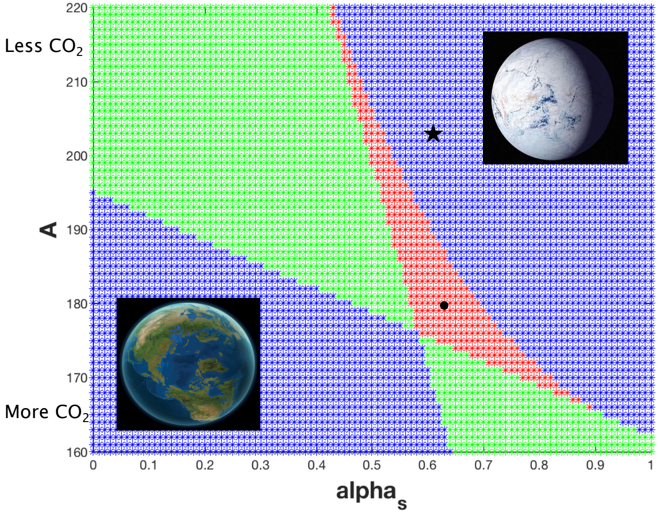

Figure: Two-dimensional bifurcation diagram indicating the number of ice line equilibria as A and α_S are varied. Red means two equilibria (one stable, one unstable); green means one equilibrium (the other root is less than 0 or greater than 1); blue indicates no equilibria. Figure by Cara Donovan.