Mathematical Models MATH 303

Homework Assignment #5

Due Fri., Nov. 9, 5:00 pm

You should write up solutions neatly to all problems, making sure to show all your work. A nonempty subset will be graded. You are encouraged to work on these problems with other classmates, and it is ok to use internet sources for help if it's absolutely necessary (with proper citation); however, the solutions you turn in should be your own work and written in your own words.

Note: Please list the names of any students or faculty who you worked with on the assignment.

1. Read Chapter 6, "Nonlinear Population Models: An Introduction to Qualitative Analysis Using Phase Planes," from the course textbook *Topics in Mathematical Modeling* by K. K. Tung.

   Complete the following exercises from Chapter 6 (pp. 108–112): **2, 3, 6, 7**

   Notes: For #6, assume that $\alpha > 0$. For #7a, plot the right-hand side of the ODE versus ϕ for the three cases. For #7d, there are two conditions, one of which depends on the initial condition $\phi(0)$. You can assume $\alpha > 0$. Check out this YouTube clip: *Synchronous Fireflies*

2. Recall the depensation growth model discussed in class, with harvesting proportional to population,

$$\frac{dP}{dt} \;=\; rP\left(1-\frac{P}{K}\right)\left(\frac{P}{P_c}-1\right) - qEP\,,$$

   where r, K, P_c, q, and E are parameters.

   (a) There are three equilibrium points: $0, P_1, P_2$ with $P_1 < P_2$. Find explicit formulas for P_1 and P_2, and show that they lie between P_c and K (when they exist).

   (b) Fixing all parameters except for E, show that a saddle-node bifurcation occurs at

$$E^* \;=\; \frac{r}{4qKP_c}\,(K-P_c)^2\;.$$

   (c) Describe the behavior of the fish population before, at, and after the saddle-node bifurcation. Draw phase lines representing each case.