

MATH 242: Principles of Analysis

Homework Assignment #8

DUE DATE: Thurs., Nov. 12, start of class.

Homework should be turned in at the BEGINNING OF CLASS. You should write up solutions neatly to all problems, making sure to show all your work. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your **own** work. Please cite any references (web based or text) that you may have used for assistance with the assignment.

Note: Please list the names of any students or faculty who you worked with on the top of the assignment.

1. Use the Intermediate Value Theorem to show that $f(x) = x^4 + 5x - 7$ has at least two real roots. Find intervals of length one in which each root must be contained.
2. A *great circle* is a circle on a sphere whose center is the same as the center of the sphere. For example, the equator is a great circle as is any circle passing through both the north and south poles of a sphere. Two points on a sphere are *antipodal* if they are diametrically opposite. For example, the north and south poles are antipodal points. Show that, at any given moment in time, there are two antipodal points with the **same** temperature on any great circle around the Earth. You may assume that the temperature function T is continuous.
3. Do the following exercises from the course text *Understanding Analysis* by Stephen Abbott: **4.3.2, 4.4.3 (which includes 3.3.1), 4.4.4, 4.5.2, 4.5.3, 4.5.7**

Notes/Hints: To complete problem **4.4.3**, you must also complete exercise **3.3.1**. For problem **4.5.7**, consider the function $g(x) = f(x) - x$.