MATH 242: Principles of Analysis

Homework Assignment #2

DUE DATE: Thurs., Sept. 17, start of class.

Homework should be turned in at the BEGINNING OF CLASS. You should write up solutions neatly to all problems, making sure to show all your work. You are strongly encouraged to work on these problems with other classmates, although the solutions you turn in should be your **own** work. Please cite any references (web based or text) that you may have used for assistance with the assignment.

Note: Please list the names of any students or faculty who you worked with on the top of the assignment.

1. Let A be any nonempty subset of $\mathbb{R}$ that is bounded below. Define the set $-A$ to be $-A = \{-a : a \in A\}$.

   a) Show that $\inf(A) = -\sup(-A)$.

   b) Use part **a)** to show that the Axiom of Completeness implies that any set of real numbers that is bounded below has a greatest lower bound.

2. Let A and B be nonempty subsets of $\mathbb{R}$ that are bounded above, and define

$$A + B = \{a + b : a \in A \text{ and } b \in B\}.$$

   Show that $\sup(A + B) = \sup(A) + \sup(B)$.

3. Given two sets A and B of $\mathbb{R}$, define the set AB to be

$$AB = \{ab : a \in A \text{ and } b \in B\}.$$

   Find an example of nonempty subsets A and B that are bounded above, but $\sup(AB) \neq \sup(A)\sup(B)$.

4. Show that $\displaystyle\bigcap_{n=1}^{\infty}[0, 1/n] = \{0\}$.

5. Do the following exercises from the course text *Understanding Analysis* by Stephen Abbott: **1.2.10, 1.3.4, 1.3.6, 1.4.2, 1.4.3, 1.4.5**.