MATH 241 Multivariable Calculus

SOLUTIONS to Final Exam Review Questions

1. (a) (iii)
   (b) (i)
   (c) (iii)
   (d) (iii)
   (e) (iii)

2. (a) $\vec{AB} =\, <1, -2, 2>, \vec{AC} =\, <2, 0, 2>, \vec{BC} =\, <1, 2, 0>$
   (b) B and C
   (c) $45°$ or $\pi/4$
   (d) $x = 5 - 4t, y = 2t, z = 1 + 4t$

3. $\mathbf{T}(1) =\, <\dfrac{1}{3}, \dfrac{2}{3}, \dfrac{2}{3}>, \mathbf{N}(1) =\, <-\dfrac{2}{3\sqrt{5}}, -\dfrac{4}{3\sqrt{5}}, \dfrac{5}{3\sqrt{5}}>$

4. (a) $<\cos t, \sin t, t>$, 3 loops
   (b) speed is $\sqrt{2}$, time to snitch is 6π
   (c) 6π meters
   (d) $9\pi/\sqrt{2}$, Gryffindor wins (of course!)

5. (a) Domain is $\mathbb{R}^2$, Range is $3 \le z \le 5$
   (b) $(0, 0)$ is a saddle point.
   (c) Level curves are hyperbolas. The x- and y-axes are level curves.

6. (a) $(0, 0), (1, 1), (-1, -1)$.
   (b) $(0, 0)$ is a saddle point, $(1, 1)$ is a local maximum, $(-1, -1)$ is a local maximum.
   (c) No absolute min, but absolute max is 0.

7. 64

8. (a) 0
   (b) $\dfrac{16\sqrt{2}\,\pi}{3}\left(2 - \sqrt{3}\right)$

9. (a) Show that $Q_x - P_y = 0$.
   (b) $f(x, y) = e^{xy} + \sin(x - y) + 3y$
   (c) $1 - 3\pi/2$

10. 0

11. (a) $0 \le r < \infty, 0 \le \theta \le \pi/2$. The value of the integral is $\pi/4$.