MATH 136-02, MATH 136-03 Calculus 2, Fall 2018

Computer Lab #3: Slope Fields

DUE DATE: Friday, December 7, 5:00 pm

The goal for this lab project is to further your understanding of the geometric and analytic concepts utilized in the study of differential equations. Since most differential equations cannot be explicitly solved, this is an important aspect of the field. You will use Maple to obtain the slope field of a given differential equation and to graph solution curves on the slope field. In the process, you should gain a better understanding of the relationship between the function defining a differential equation and the geometry of the solutions to that equation. This approach to studying differential equations is known as **qualitative analysis**.

It is **required** that you work in a group of two or three people. Any help you receive from a source other than your lab partner(s) should be acknowledged in your report. For example, a textbook, web site, another student, etc. should all be appropriately referenced. Please turn in one report per group, listing the names of the groups members at the top of your report. Be sure to answer all questions carefully, writing in complete sentences. You should **type** your answers, leaving space for mathematical calculations or graphs. You may include calculations and graphs in an appendix at the end of your report. There are exactly **four graphs** to be turned in for this project.

Slope Fields

One technique for understanding solutions to a differential equation is to draw the **slope field**. Given a differential equation

$$\frac{dy}{dx} \; = \; f(x, y) \,,$$

we plot a tiny slope segment at each point (x, y) in the plane whose slope is given by the function value at that point, $m = f(x, y)$. Using a computer, we can obtain a picture of the solution curves for the differential equation by plotting hundreds of slope marks in the xy-plane. Then, since any solution $y = y(x)$ must satisfy the ODE, each solution curve must be tangent to the slope field at every point. Imagine driving a car in a special parking lot with slope marks everywhere; you must always head in a direction tangent to the slope field at every point. The path of your car traces out a solution to the ODE.

Maple can draw slope fields using the `DEplot` command from the `DEtools` package. To begin, load this package into Maple:

```
with(DEtools):
```

You will need to execute this command every time you run a new Maple session.

To plot the slope field for the equation $\frac{dy}{dx} = x + y$, you type the following:

```
deq := diff(y(x),x) = x + y(x):
DEplot(deq,y(x),x=-3..3,y=-2..2);
```

You should obtain the slope field shown on the left in Figure 1. The first command above defines the differential equation. You can use the $\frac{d}{dx} f$ button (change f to $y(x)$) instead of `diff(y(x),x)`. However,
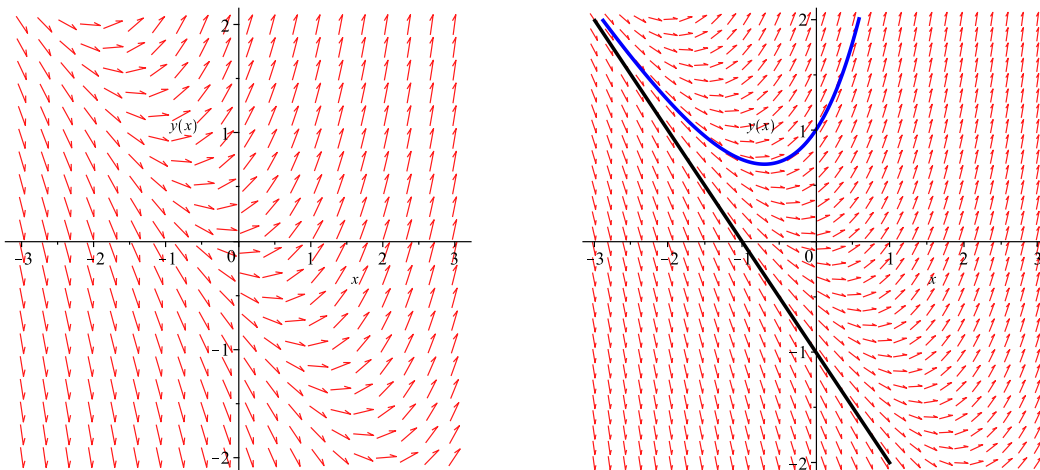
Figure 1: The slope field for $dy/dx = x+y$ (left) and the same slope field along with two solutions (right) satisfying the initial conditions $y(0) = 1$ (blue) and $y(0) = -1$ (black).

it is critical to notice that whenever the dependent variable is entered on the right-hand side, **it must be entered as y(x) rather than just plain y.** For example, to obtain $\sin y$, we type `sin(y(x))`. The `DEplot` command is used to draw the slope field of the differential equation `deq`, where the x-range is given by `x=-3..3` and the y-range is specified by `y=-3..3`. (You may want to change these depending on the differential equation.)

To draw solution curves on top of your slope field, type the following commands:

```
ics := [y(0)=1,y(0)=-1]:
DEplot(deq,y(x),x=-3..3,y=-2..2,ics,dirgrid=[30,30],stepsize=0.01,linecolor=[blue,black]);
```

This should give the slope field for $dy/dx = x+y$ and two solution curves (the first in blue, the second in black) through the points $y(0) = 1$ and $y(0) = -1$, respectively (see the right graph in Figure 1). Notice how the solution curves are tangent to the slope field everywhere.

The first command above specifies the initial conditions for the differential equation. These must be entered as a list, with brackets around the entire list. The second command contains some important options. Placing `ics` into the `DEplot` command tells Maple to draw solution curves on top of the slope field. The `dirgrid=[30,30]` option is an array of two integer values, specifying the number of horizontal and vertical mesh points to use for arrows. (So in this case we have roughly $30 \cdot 30 = 900$ little slope segments.) For this command, [2,2] is the minimum, and [20,20] is the default value. The `stepsize=0.01` option tells Maple to take steps of length 0.01 to draw the solution curves. The lower the stepsize, the better the approximation to the solution, but the longer it takes to create it. (Be careful not to crash your computer!) Finally, the `linecolor=[blue,black]` option instructs Maple to draw the two solution curves in blue and black, respectively. This makes the curves dark enough to be visible upon printing. If there were three initial conditions and hence, three solution curves, you would need to specify three colors. Play around with the values of these options so you can learn how they effect the resulting figure generated by Maple.

Be forewarned that the solutions Maple draws may not always be correct or even useful. It is not hard to introduce numerical error when approximating the solution to a differential equation. If you would like to learn more about the `DEplot` command and its options, execute the command `?DEplot` and peruse Maple's built-in help window.

Exercises:

For each of the differential equations listed below (1–4), do the following:

(a) Use Maple to draw the slope field (you choose an appropriate range). You do NOT need to turn in your plot.

(b) Plot the solution curves corresponding to the initial conditions $y(0) = 1, y(1) = 0$, and $y(0) = -1$ on the slope field. Turn in a printout with the the slope field and solution curves on the same plot.

(c) Describe the long-term behavior of each of the three solution curves from part **(b)**. In other words, what is $\lim_{x \to \infty} y(x)$ for each curve? How does it approach this value? Does it oscillate? Be as specific as possible.

(d) For differential equation #1, find the particular solution (by hand) satisfying $y(0) = 1$. For differential equation #2, find the particular solution (by hand) satisfying $y(0) = -1$. Finally, what is the solution to differential equation #4 satisfying $y(1) = 0$? Check that your answers agree with the solution curves graphed in part **(b)**.

1. $\dfrac{dy}{dx} = x^2 y^2$

2. $\dfrac{dy}{dx} = y(y - 1)$

3. $\dfrac{dy}{dx} = \cos(x^2 - 2)$

4. $\dfrac{dy}{dx} = x \sin y$