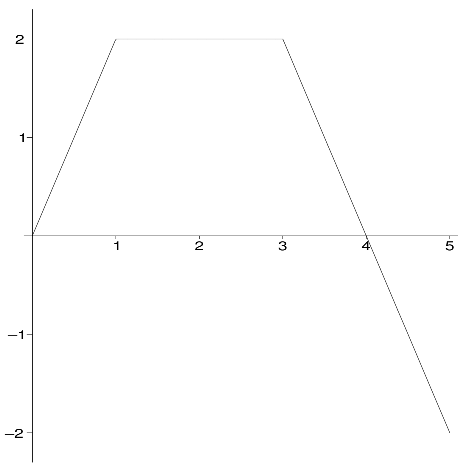
MATH 136-02, 136-03 Calculus 2, Fall 2018

Sample Final Exam Questions

Below are some **sample** final exam questions from past exams.
Note: Collectively, these are not intended to represent an actual exam nor do they completely cover all the material that could be asked on the exam.

1. Define $A(x) = \int_0^x f(t)\,dt$ for $0 \le x \le 5$, where the graph of $f(t)$ is given below.



   (a) Find $A(0)$, $A(3)$ and $A(5)$.

   (b) Find $A'(1)$ if it exists. If it does not exist, explain why.

   (c) Find $A''(1)$ if it exists. If it does not exist, explain why.

   (d) Find the intervals on which $A(x)$ is increasing and decreasing.

   (e) Find the intervals on which $A(x)$ is concave up and concave down.

   (f) Sketch a graph of $A(x)$ over the interval $0 \le x \le 5$.

2. Evaluate the following integrals:

   (a) $\displaystyle\int \sqrt{2x+1} + \sin(3x)\,dx$

   (b) $\displaystyle\int \sec^2(2\theta)\,e^{\tan(2\theta)}\,d\theta$

   (c) $\displaystyle\int t^6 \ln t\,dt$

   (d) $\displaystyle\int \frac{z^2}{\sqrt{1-z^2}}\,dz$

   (e) $\displaystyle\int \frac{6}{x(x-1)(x+1)}\,dx$

3. Approximate the value of the integral $\int_1^3 \cos(x^2)\,dx$ using the given rule:

   (a) Midpoint Rule M_4

   (b) Trapezoid Rule T_4

   (c) Simpson's Rule S_8

4. Let R be the region in the first quadrant bounded by $y = \sqrt{x}$ and $y = x^2$.

   (a) Sketch the region R and find its area.

   (b) Find the volume of the solid of revolution obtained by rotating R about the x-axis.

   (c) Find the volume of the solid of revolution obtained by rotating R about the line $x = -2$.

5. Sequences and Series:

   (a) Find a formula for the general term a_n (start with $n = 1$) for the sequence

$$-1, \frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \frac{1}{32}, - + \dots$$

   (b) Does the sequence given by $a_n = \ln\left(\frac{1+en^3}{4+n^3}\right)$ converge or diverge? If it converges, find the limit.

   (c) Find the sum of the geometric series: $18 - 6 + 2 - 2/3 + 2/9 - + \cdots$

   (d) Find the interval of convergence of the infinite series $\sum_{n=1}^{\infty} \frac{x^n}{n^2 4^n}$.

6. Determine whether the given infinite series converges or diverges using any of the tests from class or the text. You must provide a valid reason to receive full credit.

   (a) $\sum_{n=1}^{\infty} \frac{n^3}{n^4+1}$

   (b) $\sum_{n=1}^{\infty} \frac{\cos n}{n^3}$

   (c) $\sum_{n=1}^{\infty} \frac{2^n}{(2n)!}$

   (d) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$

   (e) $\sum_{n=1}^{\infty} ne^{-2n}$

7. Find the solution to the given initial-value problems:

   (a) $\dfrac{dy}{dt} = \dfrac{y}{1+t^2}, \quad y(0) = 3.$

   (b) $\dfrac{dy}{dx} = -2x^2 e^y, \quad y(3) = -\ln 9.$

8. Consider the initial-value problem

$$\frac{dy}{dt} = 2(t+1)y^2, \quad y(0) = 1/2.$$

   (a) Using Euler's method with a step-size of $\Delta t = 0.2$, approximate the value of the solution when $t = 1$. In other words, approximate $y(1)$ where $y(t)$ is the solution to the given ODE.

   (b) Solve the ODE with the given initial condition.

   (c) Using your answer to part **(b)**, calculate $y(1)$. What is the error in your approximation? (Any ideas to why it is so far off?)

9. Suppose that Auntie Pat is cooking her Thanksgiving turkey (tofurkey for you vegetarians) for friends and family. The guests are planning to arrive at 5:00 pm. She preheats the oven to 400°F. Suppose the initial temperature of the turkey is 50°F. She places the turkey in the oven at 10:00 am. By noon the turkey has cooked to a temperature of 80°F. Using Newton's law of cooling (or warming), at what time (to the nearest minute) will the temperature of the turkey be 150°F (medium rare and ready to serve)? Assume that the oven has a constant temperature of 400°F throughout the cooking. Does she make it in time for the guests or will she be serving hors d'ouvres for a while?

10. Some conceptual questions:

   (a) Derive the formula for the volume of a sphere of radius r by rotating the top half of the circle $x^2 + y^2 = r^2$ about the x-axis.

   (b) Find the average value of the function $x \sin x$ over the interval $0 \leq x \leq \pi$.

   (c) Suppose that $p(x)$ is a piecewise function defined as follows:

$$p(x) = \begin{cases} 0 & \text{if } x < 0 \text{ or } x > 2 \\ Cx^2(2-x) & \text{if } 0 \leq x \leq 2. \end{cases}$$

   Find the value of C which makes p a probability density function.

   (d) TRUE or FALSE: If true, provide a brief explanation. If false, give a counterexample to the statement.

   If $\lim_{n\to\infty} a_n = 0$, then the series $\sum_{n=1}^{\infty} a_n$ converges.

   (e) Find the value of the infinite series:

$$1 - \frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + - \cdots$$